NON-LINEAR DYNAMICS OF CABLE-STAYED MASTS

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Abstract. Cable-stayed structures are widely used to build towers and bridges, cover wide spans and in off-shore structures, among others. In this work, the non-linear finite element method, using an updated Lagrangian formulation, is used to study the non-linear vibrations of cable-stayed masts subjected to axial time dependent loads. The non-linear equations are solved using the Newton-Raphson method associated to an arc-length technique and the Newmark method is used to calculate the time responses of the system. Validation examples are presented and the influence of initial geometric imperfections and cable tensioning is studied when stayed towers are subjected to dynamic loads. Using the Budianski’s criterion, the loss of stability under sudden and harmonic loads is also analyzed. Obtained numerical results show the influence of both cable tensioning and cable positioning on the non-linear behavior of the system and could be used as a tool for an analysis of the nonlinear dynamics of the structure previous to design.

Keywords: Cable-stayed structures, Non-linear oscillations, Non-linear finite element, Dynamic instability.

1. INTRODUCTION

Cable-stayed truss and tube masts and towers are widely used in several engineering areas with applications in civil, off-shore, mechanical, telecommunications and aero-space engineering. The efficiency of these structures to support axial loads is due to the stay cables and their behavior is characterized by large displacements associated with high load bearing ratios. As cable-stayed structures show large displacements, high non-linearities are associated with their static and dynamic behavior. Therefore, the knowledge of their non-linear behavior is of interest to engineers and scientist. The cables of a cable-stayed structure work solely in tension. The cables must not only have sufficient capacity to carry the dead loads, but must also have enough reserve capacity to carry the live loads.

Due to their efficiency and different engineering applications, the analysis of cable-stayed structures has been object of several investigations in the last decades. Among the most important studies we can mention Neves (1990), who presented a finite element program to study the non-linear static and dynamic behavior of cable-stayed bridges. Using a three-dimensional model, he showed that, due to their non-linearity, the cables strongly influence the structural response of the system. Kahla (1997), using a three dimensional finite element model, studied the non-linear response of cable-stayed towers. The obtained results demonstrated that, during the non-linear response, the structure failed due to the compression forces generated by the cables. Xu et al. (1997) proposed a three dimensional finite element to study the dynamic response of the stayed towers of the Tsing Ma bridge. They showed that there is a high dynamic interaction between towers and cables which affects the natural frequencies of the system. Wahba et al. (1998a, 1998b), using the nonlinear finite element method and an analytical model, studied the non-linear static and dynamic response of stayed towers. Obtained results showed that the analytical model presents lower displacements if compared with those obtained by the finite element method. Experimental models were also studied and reliable agreement between numerical the model was observed.

Kahla (2000) studied the effect of cable failure on the dynamic response of stayed towers concluding that, if the failure occurs in certain cables, the chance of failure of the whole structure is increased. Millar and Barghian (2000), using two finite element codes, studied the static and dynamic response of structures that displays dynamic jumps. They concluded that non-linear static problems can be analyzed as dynamic systems without dumping. Cheng et al. (2002), using an advanced non-linear finite element formulation, studied the aerostatic stability of stayed bridges showing the high non-linearity of the response due to lateral winds. Chan et al. (2002) performed a second order analysis of imperfect stayed columns and showed that the buckling load of the columns can be increased by the pre-tensioning of the stay cables.

Yan-Li et al. (2003), using a discrete model, analyzed the vibrations of stayed masts under wind loads, finding a good agreement between experimental and numerical results. Freire et al. (2005) studied the non-linear effects of a stayed bridge using both linear and non-linear finite elements. The obtained results show that the cable curvature increases the non-linearity of the system mainly when large displacements generate axial tensions on the cables. Pasquetti (2003) studied the buckling and vibration characteristics of cable-stayed towers using a simplified sdof model. Finally, Orlando (2006) studied the non-linear dynamics and control of tower-pendulum system under harmonic loads. A detailed parametric analysis of the non-linear oscillations showed that a non-linear pendulum absorber can increase or decrease the vibration amplitudes of the tower.
In this work, the non-linear finite element method, using an updated Lagrangian formulation, is used to study the non-linear vibrations of cable-stayed masts subjected to axial time dependent loads. The non-linear equations are solved using the Newton-Raphson method associated to an arc-length technique and the Newmark method is used to calculate the time responses of the system. Validation examples are presented and the influence of initial cable tensioning is studied when stayed towers are subjected to dynamic loads. Using the Budianski’s criterion, the loss of stability under sudden and harmonic loads is also analyzed. Obtained numerical results show the great influence of both cable tensioning and cable positioning on the non-linear behavior of the system.

2. MATHEMATICAL FORMULATION

The present formulation is based on previous works by Silveira (1995), Galvão (2000), Oliveira (2002), Campos Filho (2004) and Carvalho (2008) who implemented finite element models to analyze the geometric non-linear behavior of plane structural frames and cable system. This formulation is based on the strain-displacement relations using a complete Green-Lagrange tensor. The column is modeled using beam-column elements and the cables, using truss elements. The material of all elements is considered to be linear and elastic.

3. NUMERICAL RESULTS

Consider a perfect clamped-free column with internal diameter = 0.475 m, external diameter = 0.500 m and elasticity modulus $E=1.18\times10^8$ kN/m$^2$, respectively. The column is clamped at the base and supported by two inclined cables with $\alpha=60^\circ$, cross-section diameter = 0.018 m, elasticity modulus $E=1.0\times10^8$ kN/m$^2$ and a pre-tensioning force $T=10$ kN. The column is subjected to an axial load $P$ and a perturbing moment $M$ as shown in Fig. 1(a).

Figure 1(b) shows the influence of stay cables on the post-critical behavior of an axially loaded column. When no cables are considered in the analysis, after the critical load, the system displays a stable post-critical path with a small initial curvature. If the two cables are considered, the value of the critical load increases more than seven times, but the system displays in this case an unstable post-buckling behavior with a sharp decrease in the load carrying capacity, being the minimum post-critical load, associated with a fold bifurcation, lower than the critical load of the column without cables. So, when the column reaches the critical loads it jumps to a post-buckling configuration associated with large displacements. Figure 1(c) displays the deformed configuration of the cable-stayed for selected equilibrium positions along the non-linear post-buckling path identified by capital letters in Fig. 1(b). So, based on the theory of elastic stability, a high imperfection sensitivity is expected for this structural system.

Figure 2 shows the effect of initial cable tensioning on the post-buckling paths of the axially loaded column. If different tensioning is considered, say $T_1=9$ kN and $T_2=10$ kN, initial geometric imperfections on the column are induced. Figure 2(a) displays the initial deformed configuration of the column due to different initial cable tensioning and the geometric and load parameters used in the analysis. Figure 2(b) displays the variation of the lateral displacement of the top of the column, $v$, with the load parameter, while figure 2(c) shows the variation of the axial displacement $u$. When compared with the perfect case (equal cable tensioning) the nonlinear equilibrium path shows a rather different behavior. The column looses stability at a limit point which is much lower than the critical load of the perfect system. The decrease in the critical load is of about 60%. This illustrates the high imperfection sensitivity of this structural system and the deleterious effect of asymmetric cable tensioning on the non-linear response.
Figure 2. Imperfect cable-stayed column. (a) Geometric characteristics. (b) Post-critical paths - vertical displacements (c) Post-critical paths - lateral displacements

Figure 3. Influence of number of cables and inclination angle on the natural frequency. (a) Column with two cables. (b) Column with four cables. (c) Variation of natural frequency.

Figure 3 shows the influence of the number and inclination of the stay cables on the lowest natural frequencies of the column. Two configurations are analyzed, considering two and four stay cables, as shown in Figures 3(a) and 3(b) respectively. In both cases the height of the attaching point remains constant and we vary the distance of the cable support to the column, thus varying the inclination angle, $\alpha$. The results show that these parameters have an important influence on the lowest natural frequency. For the column with two cables, as the value of angle $\alpha$ increases, the natural frequency increases reaching a maximum value at $\alpha=40^\circ$, after this value, as the $\alpha$ angle increases, the natural frequency is reduced tending to zero for $\alpha=90^\circ$. When four stay cables are considered, the maximum frequency occurs at $\alpha=45^\circ$. Comparing both curves, it is possible to observe that the natural frequency increases as the number of stay cables increase.

Now consider the column with two stay cables and subjected to a suddenly applied axial load and a harmonic axial load with frequency $\Omega$ as illustrated in Fig. 4.
Consider initially the cable-stayed column of Fig. 4(a), subjected to a suddenly applied axial load $P$. Fig. 5 displays the time responses of the stayed column for increasing values of axial load. Zero initial conditions are considered in the analysis. As shown in Fig. 5(a), for certain values of $P$, the damped response converge to a pre-buckling static configuration. As $P$ increases, see Figs. 5(b) and 5(c), the column displays increasing vibration amplitudes and jumps to a post-buckling configuration. Figure 6 shows the loss of stability of the cable-stayed column for increasing values of axial load, using the Budianski’s criterion. The critical load is $P/P_{cr} \approx 0.95$.

Fig. 4(b) shows the column subjected to an axial harmonic load $P$ with amplitude $P_a$ and frequency $\Omega$. Fig. 7 shows the lateral time responses of the stayed column, considering a forcing frequency equals to the natural frequency ($\omega_0$) of the system and increasing values of the forcing amplitude. As shown in Figs. 7(a) and 7(b), for small values of the forcing amplitude, the column displays small amplitude lateral oscillations. However, in Fig. 7(c), for $P_a = 2.40P_{cr}$, the lateral displacement of the column grows exponentially, indicating loss of instability of the Mathieu type.

Figure 8 shows the variation of the maximum lateral displacement of the column as a function of the forcing amplitude. In Fig. 8(a) the forcing frequency $\Omega = \omega_0$ while in Fig. 8(b), $\Omega = 2\omega_0$. For $\Omega = \omega_0$, the critical load, according with Budianski’s criterion, $P/P_{cr} \approx 2.30$. For $\Omega = 2\omega_0$, $P/P_{cr} \approx 0.70$. This case corresponds to the main parametric instability region, being the dynamic buckling load much lower than the static critical load.
4. CONCLUSIONS

In this work, the non-linear finite element method, using an updated Lagrangian formulation, is employed to study the non-linear vibrations of cable-stayed masts subjected to axial time dependent loads. The non-linear equations are solved using the Newton-Raphson method associated to an arc-length technique and the Newmark method is used to calculate the time responses of the system.

Using the Budianski’s criterion, the loss of stability under sudden and harmonic loads is also analyzed. As observed, the behavior of the system is highly influenced by cable tensioning and load characteristics, which generates lower or
higher instability loads. Then numerical results show the great influence of both cable tensioning and cable positioning on the non-linear behavior of the system.

5. ACKNOWLEDGEMENTS

This work was made possible by the support of the Brazilian Ministry of Education – CNPq and FAPERJ-CNE.

6. REFERENCES

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