NONLINEAR ANALYSIS OF THE RESPONSE OF AN AEROELASTIC SYSTEM USING SHAPE MEMORY ALLOYS

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Abstract. In this paper, a theoritical simulation study of the nonlinear response of a two degree of freedom typical airfoil section using a Shape Memory Alloy (SMA) is presented. The model is integrated into a numerical solution of the aeroelastic nonlinear dynamic system that results from the inclusion of Shape Memory Alloy components in a dynamic structural system. The objective of the present work is to obtain bifurcation diagrams of a two degree of freedom airfoil subjected to two-dimensional incompressible flow taking into account structural nonlinearities, where the simulations are investigated employing a numerically refined approach. In the present investigation, concentrated the SMA nonlinearities in the airfoil pitch.

The integro-differential aeroelastic equations of motion for the two degree of freedom airfoil are reformulated into a system of eight first-order autonomous ordinary differential equations. The term bifurcation is used to describe qualitative changes that occur in the orbit structure of a system, as a consequence of parameter changes. Numerical simulations show that the coupled nonlinearities can generate a variety of motions

Keywords: Aeroelasticity, Shape Memory Alloy, Nonlinear Dynamic, Bifurcation

1. INTRODUCTION

Shape Memory Alloys (SMA) consist of a group of metallic materials that demonstrate the ability to return to some previously defined shape when they subjected to the appropriate thermal procedure. The shape memory effect occurs due to a temperature and stress dependent shift in the materials crystalline structure between two different phases called martensite and austenite. Martensite, the low temperature phase, is relatively soft whereas austenite, the high temperature phase, is relatively hard. SMAs have been used in a variety of applications. The dynamical response of the shape memory systems is introduced in different references, (Savi et. al, 2008), (Piccirillo et. al., 2009).

Aeroelasticity is the dynamic interaction of structural, inertial, and aerodynamic forces. Conventional methods of examining aeroelastic behavior have relied on a linear approximation of the governing equations which describe both the flow field and the structure. The nonlinear aeroelastic systems may exhibit nonlinear dynamic response characteristics such as limit cycle oscillations (LCOs), internal resonances, and chaotic motion, (Lee, et.al., 1999).

More recently, aeroelastic modeling has considered the combination of nonlinear and stochastic responses via the inclusion of the effects due to flow random perturbations, as done in (Poirel and Price, 2001). In Dessi and Mastroddi (2008), is analyzed the performed on a simplified aeroelastic model retaining only two structural modes (first bending and first torsional modes) and with a simplified description of both unsteady loads due to wing oscillation and external gust excitation.

Indeed, the inclusion of vertical gust effects in the aeroelastic modeling provides the physical mechanism by which the wing is actually perturbed in the rest condition. This phenomenon has recently been investigated experimentally providing new insight about how the forced wing response combines with the potential onset of LCO in certain flow speed regimes (Tang et al., 2000; Tang and Dowell, 2002). In particular, in the knee-bifurcation scenario, a vertical gust of adequate intensity might induce LCOs of relevant amplitude even below the linear flutter speed.

In Tang et. al. (2004) show a theoretical simulation study of the non-linear gust response of a three degree-of-freedom typical airfoil section with a control surface using an electromagnetic dry friction damper. In this work, we consider the effect of gust loads on the wing section. Sinusoidal and linear frequency sweep gust loads are used. The present results may be helpful in better understanding physically the alleviation of a typical airfoil section response due to gust loads using a shape memory alloy (SMA).

2. EQUATION OF AEROFOIL MOTIONS AND GUST LOAD

Figure 1 shows a sketch of a two-degree-of-freedom (2-dof) airfoil motion in plunge and pitch. The plunge deflection is denoted by h, positive in the downward direction, and α is the pitch angle about the elastic axis, positive
nose up. The elastic axis is located at a distance \(a_b\) from the mid-chord, while the mass centre is located at a distance \(x_a\) from the elastic axis, where \(b\) is the airfoil semi-chord. Both distances are positive when measured towards the trailing edge of the airfoil.

![Figure 1: Schematic of airfoil with 2 d.o.f motion](image)

The aeroelastic equations of motion for linear springs have been derived by Fung (1969). For nonlinear SMA restoring forces with subsonic aerodynamics, the coupled bending-torsion equations for the airfoil can be written as follows:

\[
\begin{align*}
\text{Eq. 4: } & mh'' + S_a \alpha'' + K_b h = -C_L + P(t) \\
\text{Eq. 5: } & S_a h'' + I_a \alpha'' + K_a \alpha + K_{SMA}(\alpha, T) = C_M + Q(t)
\end{align*}
\]

where the symbols \(m\), \(S_a\) and \(I_a\) are the airfoil mass, airfoil static moment about the elastic axis, wing mass moment of inertia about elastic axis, respectively. \(K_b\) and \(K_a\) are the linear plunge and pitch stiffness terms, and \(C_L\) and \(C_M\) are the forces and moments acting on the airfoil, respectively and \(P(t)\) and \(Q(t)\) are the lift and pitch moments due to the gust profile, respectively. In this paper we use the polynomial model, and assuming that Eq. 3 is valid for the pure shear stress – strain behavior, Savi and Braga (1993). Note that the restitution force may be expressed as \(K_{SMA} = A\sigma\), where \(A\) is the area of this SMA element and \(\sigma = q(T - T_M) \varepsilon - b \varepsilon^3 + \frac{b^2}{4q(T_A - T_M)} \varepsilon^5\). According to Paiva and Savi (2006) the polynomial model represents in a qualitatively coherent way both martensite detwinning process and pseudoelasticity, although it does not consider twinned martensite (\(M\)). In other words, there is no stable phase for \(T < T_M\) in a stress-free state, but the authors believe that this analysis is useful to the understanding of the nonlinear dynamics of shape memory systems. The proposed model captures itself all of the essential features of the studied phenomenon.

Defining \(\tau = \frac{U}{b}, \xi = \frac{h}{b}, x_a = \frac{S_a}{b m}, \omega_a^2 = \frac{K_a}{I_a}, \omega_a^2 = \frac{K_a}{I_a} + \frac{aA(T - T_M)}{I_a}, r_a^2 = \frac{I_a}{mb^2} \mu = \frac{m}{\rho b^2 \pi}, \beta = \frac{bA}{I_a \omega_a^2}, \lambda = \frac{eA}{I_a \omega_a^2}\). The equations (4) and (5) can be written in nondimensional form as follow

\[
\begin{align*}
\frac{x_a}{r_a} \ddot{\xi} + \dot{\alpha} + \left(\frac{\Omega}{V}\right)^2 \xi &= \frac{1}{\mu \omega_a} C_L(\tau) + \frac{P(t)}{m \mu V^2} \\
\frac{x_a}{r_a} \ddot{\xi} + \dot{\alpha} + \frac{1}{V^2} \left(\alpha - \beta x_a^3 + \lambda \alpha^5\right) &= \frac{2}{\pi \mu \omega_a} C_M(\tau) + \frac{Q(t)}{m \mu V^2 I_a^2}
\end{align*}
\]

In equations (6) and (7), \(V\) is a nondimensional velocity defined as \(V = \frac{U}{b \omega_a}\) and \(\Omega = \frac{\omega_a}{\omega_a}\), where \(\omega_a\) and \(\omega_a\) are the uncoupled plunging and pitching mode natural frequencies, respectively, \(U\) is the freestream velocity, and the dot denotes differentiation with respect to the non-dimensional time \(\tau\) defined as \(\tau = \frac{Ut}{b}\), where
and \( \mu = \frac{m}{\rho b^2 \pi} \) is the airfoil/air mass ratio. For incompressible flow, Fung (1969) gives the following expressions for \( C_L(\tau) \) and \( C_M(\tau) \).

\[
\begin{align*}
C_L(\tau) &= \pi \left( \xi + a_h \ddot{u} + \dot{a} \right) + 2\pi \left[ a(0) + \xi(0) + \frac{1}{2} a_h \right] \dot{a} \phi(\tau) + 2\pi \int_0^1 \phi(\tau - \sigma) \left( \ddot{a}(\sigma) + \frac{1}{2} a_h \right) \ddot{a} \, d\sigma \quad (10) \\
C_M(\tau) &= \pi \left[ \frac{1}{2} a_h \right] \left[ a(0) + \xi(0) + \frac{1}{2} a_h \right] \dot{a} \phi(\tau) + \pi \left[ \frac{1}{2} a_h \right] \int_0^1 \phi(\tau - \sigma) \left( \ddot{a}(\sigma) + \frac{1}{2} a_h \right) \ddot{a} \, d\sigma
\end{align*}
\]

where the Wagner function \( \phi(\tau) \) is given by \( \phi(\tau) = 1 - \psi_1 e^{-\epsilon_1 \tau} - \psi_2 e^{-\epsilon_2 \tau} \), and the constants \( \psi_1 = 0.165 \), \( \psi_2 = 0.335 \), \( \epsilon_1 = 0.0455 \) and \( \epsilon_2 = 0.3 \) are obtained from Jones (1940).

Atmospheric turbulence creates a gust load that can be represented by two different mathematical descriptions. One is associated with a discrete gust representation usually of a deterministic nature. In the present work, a simple but enlightening hypothesis for the atmospheric (deterministic) sinusoidal gust is used. In this case, we write

\[
P(\tau) = P_0 \sin(\omega \tau) \quad \text{and} \quad Q(\tau) = Q_0 \sin(\omega_1 \tau)
\]

and let \( F = \frac{P_0 b}{m V^2} \) and \( F_1 = \frac{Q_0}{m V^2 \omega_1^2} \), so that the second term in equation (8) for \( P_1(\tau) \) becomes \( F \sin(\omega \tau) \) and in equation (9) becomes \( F_1 \sin(\omega_1 \tau) \). Note that the amplitude of gust excitation depends on the aerodynamic and airfoil parameters.

Due to the presence of the integral terms in the integro-differential equations (10) and (11), it is cumbersome to integrate them numerically. A set of simpler equations was derived by Lee et al. (1999), and they introduced four new variables

\[
\begin{align*}
w_1 &= \int_0^\tau e^{-\epsilon_1 (\tau - \sigma)} a(\sigma) \, d\sigma \\
w_2 &= \int_0^\tau e^{-\epsilon_2 (\tau - \sigma)} a(\sigma) \, d\sigma \\
w_3 &= \int_0^\tau e^{-\epsilon_1 (\tau - \sigma)} \xi(\sigma) \, d\sigma \\
w_4 &= \int_0^\tau e^{-\epsilon_2 (\tau - \sigma)} \xi(\sigma) \, d\sigma
\end{align*}
\]

equations (6) and (7) can be written as

\[
\begin{align*}
c_0 \ddot{\xi} + c_1 \dot{\xi} + c_2 + c_3 \dot{a} + c_4 \dot{a} + c_5 a + c_6 w_1 + c_7 w_2 + c_8 w_3 + c_9 w_4 &= P_1(\tau) \\
d_0 \ddot{\xi} + d_1 \dot{\xi} + d_2 \dot{a} + d_3 \dot{a} + d_4 \dot{a}^3 + d_5 \dot{a}^2 + d_6 \dot{a} + d_7 \xi + d_8 w_1 + d_9 w_2 + d_{10} w_3 + d_{11} w_4 &= Q_1(\tau)
\end{align*}
\]

where \( P_1(\tau) \) and \( Q_1(\tau) \) are functions depending on initial conditions, Wagner's function and the forcing terms, namely,

\[
\begin{align*}
P_1(\tau) &= \frac{1}{\mu} \left( 0.5 - a_h \right) a(0) + \xi(0) \right) \left( \psi_1 e_1 e^{-\epsilon_1 \tau} + \psi_2 e_2 e^{-\epsilon_2 \tau} \right) + F \sin(\omega \tau) \\
Q_1(\tau) &= -\left( 1 + 2a_h \right) P_1(\tau) + F_1 \sin(\omega_1 \tau)
\end{align*}
\]

The resulting set of eight first-order ordinary differential equations by a suitable transformation is given a

\[
\frac{dX}{d\tau} = f(X, \tau), \quad \text{where} \quad X = \{x_1, x_2, \ldots, x_8\} = \{a, \dot{a}, \ddot{a}, \dot{\xi}, \ddot{\xi}, w_1, w_2, w_3, w_4\} \in \mathbb{R}^8.
\]

Explicitly, the system can be written as
\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{c_d d_0 - c_d d_1}(c_0 E - d_0 F) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{1}{c_d d_0 - c_d d_1}(-c_1 E + d_1 F) \\
\dot{x}_5 &= x_1 - e_1 x_5 \\
\dot{x}_6 &= x_1 - e_2 x_6 \\
\dot{x}_7 &= x_3 - e_1 x_7 \\
\dot{x}_8 &= x_3 - e_2 x_8
\end{align*}
\]

where

\[
E = -Q_1(t) + d_1 x_2 + d_3 x_1 + d_4 x_1^2 + d_5 x_1^3 + d_6 x_4 + d_7 x_3 + d_8 x_5 + d_9 x_6 + d_{10} x_7 + d_{11} x_8
\]

\[
F = P_1(t) + c_2 x_1 + c_3 x_2 + c_4 x_3 + c_5 x_1 + c_6 x_5 + c_7 x_7 + c_8 x_7 + c_9 x_8
\]

and

\[
\begin{align*}
c_0 &= 1 + \frac{1}{\mu}, c_1 = x_a - \frac{a}{\mu} c_2 = \frac{2}{\mu}(1 - \psi_1 - \psi_2), c_3 = \frac{1 + 2(0.5 - a_h)(1 - \psi_1 - \psi_2)}{\mu}, c_4 = \left(\frac{\Omega}{V}\right)^2 + \frac{2}{\mu}(\psi_1 \psi_1 + \psi_2 \psi_2), \\
c_5 &= \frac{2}{\mu} \left((1 - \psi_1 - \psi_2) + \frac{d_1}{e_1} (\psi_1 \psi_1 + \psi_2 \psi_2)\right) e_6 = \frac{2}{\mu} \psi_1 \psi_1 \left(1 - (0.5 - a_h) e_1\right), c_7 = \frac{2}{\mu} \psi_2 \psi_2 \left(1 - (0.5 - a_h) e_2\right), \\
c_8 &= -\frac{2}{\mu} \psi_2^2 e_3, c_9 = -\frac{2}{\mu} \psi_2^2 e_3
\end{align*}
\]

\[
\begin{align*}
d_0 &= \frac{x_a}{\mu^2} - \frac{a}{\mu^2} c_2, d_1 = 1 + \frac{1 + 8 a_h^2}{8 \mu^2} c_2, d_2 = 1 - 2 a_h - \frac{(1 + 2 a_h)(1 - \psi_1 - \psi_2)}{2 \mu^2}, \\
d_3 &= \frac{1}{V^2} - \frac{1 + 2 a_h^2}{2 \mu^2 - \frac{(1 + 2 a_h)(1 - \psi_1 - \psi_2)}{2 \mu^2}}, d_4 = -\frac{\beta}{V^2}, d_5 = \frac{\lambda}{V^2}, d_6 = -\frac{(1 + 2 a_h)(1 - \psi_1 - \psi_2)}{2 \mu^2}, \\
d_7 &= -\frac{(1 + 2 a_h)(\psi_1 \psi_2 + \psi_2 \psi_2)}{2 \mu^2}, d_8 = -\frac{(1 + 2 a_h) \psi_1 \psi_1 \left(1 - (0.5 - a_h) e_1\right)}{2 \mu^2}, d_9 = -\frac{(1 + 2 a_h) \psi_2 \psi_2 \left(1 - (0.5 - a_h) e_2\right)}{2 \mu^2}, \\
d_{10} &= \frac{(1 + 2 a_h) \psi_2^2 e_3}{2 \mu^2}, d_{11} = \frac{(1 + 2 a_h) \psi_2^2 e_3}{2 \mu^2}
\end{align*}
\]

5. NUMERICAL ANALYSIS

Usually, the physical mechanism that makes the wing vibrate is due to airplane maneuvering and/or to gust occurrence. The numerical simulations are obtained by a fourth-order Runge-Kutta algorithm. Now, we introduce the SMA spring in pitch d.o.f. This model is used to investigate the effect of the nonlinearity in this aeroelastic system. In this case we used a temperature alloy around \( T = 323K \) approximately \( T = 50^\circ C \). The parameters of aerofoil used in this work are described in Tang et al. (2004) and present in table 1.

Table 1. System parameters of two-dimensional typical section model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span (l)</td>
<td>0.52 m</td>
</tr>
<tr>
<td>Semi-chord (b)</td>
<td>0.127</td>
</tr>
<tr>
<td>Elastic axis (a)</td>
<td>-0.0625 m</td>
</tr>
<tr>
<td>Mass of wing</td>
<td>0.713 kg</td>
</tr>
<tr>
<td>( I_a ) (per span)</td>
<td>0.0185 kg m</td>
</tr>
<tr>
<td>( S_a ) (per span)</td>
<td>0.0726 kg</td>
</tr>
<tr>
<td>( K_a ) (per span)</td>
<td>42.8 kg m/s²</td>
</tr>
<tr>
<td>( K_i ) (per span)</td>
<td>2755.4 kg m / s²</td>
</tr>
</tbody>
</table>
The shape memory alloys present different properties, depending on the temperature. In this paper, we present a study on the pseudoelastic dynamic behavior, considering a higher temperature, where austenitic phase is stable in the alloy. In all simulations, we analyze the behavior of the aeroelastic dynamical system, where the spring is assumed to be made of a Ni-Ti alloy and the properties are present in Table 2 (Paiva and Savi, 2006).

Table 2. Material constants for a Ni-Ti alloy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td>MPa/K</td>
<td>1000</td>
</tr>
<tr>
<td>b</td>
<td>Mpa</td>
<td>$40 \times 10^6$</td>
</tr>
<tr>
<td>$T_M$</td>
<td>K</td>
<td>287</td>
</tr>
<tr>
<td>$T_A$</td>
<td>K</td>
<td>313</td>
</tr>
</tbody>
</table>

We assume a sinusoidal gust and without loss in generality, the gust is only applied in the plunge degree of freedom. In this case $Q(\tau) = 0$. In addition to the sensitive dependence on the initial conditions, a dynamical system is very sensitive to small variations in the control parameters. As a control parameter varies, the stability of a dynamical system can be change, due to local or a global bifurcation. The bifurcation diagram provides a general view of the system dynamics, by plotting a system variable, as a function of a control parameter, (Alligood et. al., 1996). Fig.2 shows a global view of the bifurcation diagram of the SMA nonlinear aeroelastic model described by Eq. (14). Ranging the velocity parameter, the bifurcation diagram Fig. 2 for the plots the asymptotic values of the Poincaré points of the system variable $y$, where the transient has been omitted. In fig. 2a the bifurcation diagram shows that there is a change in the amplitude of pitch motion when the velocity is increased, and at $V = 24$ the response undergoes a period doubling bifurcation and after this bifurcation the system quickly becomes unstable. These bifurcations repeat, over and over, making a period doubling cascade, which is an infinite sequence of period doubling bifurcations up to chaos. It eventually restabilizes into a period-four oscillation at $V = 27$. A bifurcation diagram of plunge motion as a function of the velocity is shown fig. 2b. In fig. 2b shows that the system undergoes two period doubling bifurcations, the first at $V = 24$ and the second at $V = 25$. Furthermore, a region of possibly chaotic motion can be seen in the approximate range $25 < V < 27$.

Figure 2: Bifurcation diagram of the airfoil for: a) pitch motion and b) plunge motions

Fig.3 shows a global view of the bifurcation diagram of the SMA nonlinear aeroelastic model described by Eq. (14), considering the $P_0$ control parameter. By considering the bifurcation diagram at the interval, $5 \leq P_0 \leq 15$, it is possible to observe regions of cloud of points, associated with chaotic behavior. Periodic windows separate these chaotic regions (Fig. 3a). The significance of the bifurcation diagram is as follows. If for a given values of $P_0$ the system is stable, then a single point is obtained, e.g., $6 < P_0 < 7.5$, approximately. If the motion is an LCO with one frequency, then, two points are obtained ($7.5 < P_0 < 12$ approximately), and an LCO with two frequencies result in four points, etc. However, for some velocities, a large number of points are obtained (giving what appears to be almost a vertical line on the bifurcation diagram) indicating chaos behavior. The bifurcation diagram shown in Fig. 3a suggests that the route to chaos is via period-doubling. In fig 3b, the system presents only the periodic motion.
6. CONCLUSIONS

The aeroelastic behavior of a two-degree-of-freedom typical airfoil section placed in a subsonic flow with SMA nonlinearity in pitch has been investigated using an RK method. A mixed region of periodic and quasi-periodic motions dominates the bifurcation diagram, where the system’s behavior switches irregularly between periodic and quasi-periodic motions. In this paper we have found that there is a period-doubling route to chaos with the velocity increasing and also with the amplitude of the gust increasing as in pitch as plunge motion.

7. ACKNOWLEDGEMENTS

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8. REFERENCES


9. RESPONSIBILITY NOTICE

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