MODELING FAILURE OF HETEROGENEOUS VISCOELASTIC MATERIALS UNDER IMPACT USING A TWO-WAY COUPLED MULTISCALE MODEL

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Abstract. This paper presents a model for predicting damage evolution in heterogeneous viscoelastic materials under impact loading. A two-way coupled multiscale approach is employed and damage is considered in the form of multiple cracks evolving in the local (micro) scale. The objective of such a model is to develop the ability to consider energy dissipation due to both bulk dissipation and the development of multiple cracks occurring on multiple length and time scales. While predictions of these events may seem extraordinarily costly and complex, there are multiple structural applications where effective models would save considerable expense. In some applications, such as protective devices, viscoelastic materials may be preferred because of the considerable amount of energy dissipated in the bulk as well as in the fracture process. In such applications, experimentally based design methodologies are extremely costly, therefore suggesting the need for improved models. In this paper, the authors focus on the application of the developed two-way coupled multiscale model to the solution of some example problems involving impact loading of viscoelastic heterogeneous materials with growing cracks at the local scale.

Keywords: Multiscale model, Heterogeneous viscoelastic media, Impact loading, Microcracking

1. INTRODUCTION

It has been observed that many composite materials exhibit inelastic behavior when subjected to impact loading conditions. This inelastic behavior is induced by energy dissipation mostly due to the development of multiple microcracks and the viscoelastic behavior of the matrix. In inelastic media, the problem of crack growth is particularly complicated by the fact that there are at least two competing energy mechanisms that occur essentially simultaneously whenever one or more cracks run: bulk material dissipation; and fracture energy. Furthermore, due to material heterogeneities that can occur on multiple length scales, cracks can coexist over a broad range of length scales simultaneously, and the energy dissipated on these differing length scales can nevertheless be of the same order of magnitude. In some applications, such as protective devices, viscoelastic materials may be preferred because of the considerable amount of energy dissipated in the bulk as well as in the fracture process.

Because the length scales of the structural part and the microstructure of the composite are very far apart, it is not practical to consider the material heterogeneity in a single scale model. On the other hand, multiscale models are able to efficiently solve this kind of problem in an approximate fashion. The main approximations introduced in multiscale models are the assumption of the existence of a Representative Volume Element (RVE) and the use of averaging theorems. However, if one can determine the RVE for a particular composite material and the length scales of the problem are far apart, the errors introduced by the assumptions become negligible.

Even though multiscale models are more computationally efficient than single scale models, they can still be very costly depending mostly on the size and geometry of the RVE. However, due to the development of high speed computers and the advent of parallel programming, it is now possible to solve two- or three-dimensional problems on two or even three length scales. These solutions are obtained by using standard time marching multi-scale algorithms, linked by appropriate homogenization theorems (Eshelby, 1957; Hill, 1963; Hashin, 1964; Hill, 1965a,b; Allen and Yoon, 1998), which are able to predict the evolution of hundreds, even thousands of cracks simultaneously. In this paper, an example solution for a viscoelastic media with local microcracks is given in order to demonstrate the capabilities of the model.

2. MULTISCALE MODEL

The main objective of multiscale models is to determine the overall constitutive behavior of heterogeneous materials simultaneously throughout the analysis based on the behavior of the individual constituents and their interactions at the local scale. Multiscale models become very attractive for problems with evolving microstructure due to formation and growth of microcracks, since the evolution of the microstructure is necessarily both spatially and time dependent, otherwise the classical homogenization technique, where average properties of the heterogeneous material are determined a priori, may be sufficient and less time consuming.

Figure 1 schematically presents a structural part which is statistically homogeneous at the global scale but heterogeneous at the local scale, where the microstructure may contain inclusions as well as growing microcracks. In Figure 1,
superscripts refer to a scale index, where 0 refers to the global scale and 1 refers to the local scale, $V^\mu$, $\partial V^\mu_0$ and $\partial V^\mu_1$ are the volume, external and internal boundary surfaces of scale $\mu$, respectively, $\ell^\mu$ is the length scale associated with scale $\mu$ and $\ell^\mu_c$ is the length scale associated with the cracks at scale $\mu$.

Figure 1: Description of a three scale problem

Note that the cohesive zone ahead of crack tips may itself be considered a third scale in the problem. However, crack propagation is herein modeled by using a cohesive zone model. More specifically, we use the micromechanical viscoelastic cohesive zone model developed by Allen and Searcy (2001a). Allen and Searcy (2001a,b) have developed a cohesive zone model for viscoelastic media that is inherently two scale in nature in that it utilizes the solution to a microscale scale continuum mechanics problem, together with a homogenization theorem to produce a cohesive zone model on the next larger length scale.

Therefore, posing the Initial Boundary Value Problem (IBVP) for the global object, gives:

$$\sigma^0_{ij,j} = \rho^0 \frac{d^2 u^0_i}{dt^2} \text{ in } V^0$$  \hspace{1cm} (1)

where $\sigma^0_{ij}$ is the Cauchy stress tensor, $\rho^0$ is the mass density of the statistically homogeneous object, $u^0_i$ is the displacement vector and $V^0$ is the volume of the object at the global length scale. Note that body forces have been neglected in Eq. 1.

$$\sigma^0_{ij} = \sigma^0_{ji} \text{ in } V^0$$  \hspace{1cm} (2)

$$\varepsilon^0_{ij} = \frac{1}{2} (u^0_{i,j} + u^0_{j,i}) \text{ in } V^0$$  \hspace{1cm} (3)

where $\varepsilon^0_{ij}$ is the infinitesimal strain tensor defined on the global length scale.

$$\sigma_{ij}^1(t) = \Omega_{\tau=\infty}^{\tau=t}{\varepsilon^1_{ij}(\tau)} \text{ in } V^1$$  \hspace{1cm} (4)

where $\Omega_{\tau=\infty}^{\tau=t}$ is a functional mapping that accounts for history dependent effects, such as viscoelasticity, and is determined by locally averaging the response at the local scale. Note that Eq. 4 is not known a priori but can be determined during the multiscale analysis.

Now considering that continuum mechanics still applies at the microscale, assuming that the global length scale is much larger than the local length scale, $\ell^0 \gg \ell^1$, that the length scale associated with cracks at the local scale is much smaller than the local length scale, $\ell^1 \gg \ell^1_c$, that these cracks are homogeneously distributed at the local scale, and finally, that the length of the wave propagating on the global scale, $\ell^0_w$, is much larger than the local scale length, $\ell^0_w \gg \ell^1$, the local IBVP can be approximated as a quasi-static problem given by:

$$\sigma_{ij}^1 = 0 \text{ in } V^1$$  \hspace{1cm} (5)

$$\sigma_{ij}^1 = \sigma_{ji}^1 \text{ in } V^1$$  \hspace{1cm} (6)

$$\varepsilon_{ij}^1 = \frac{1}{2} (u_{i,j}^1 + u_{j,i}^1) \text{ in } V^1$$  \hspace{1cm} (7)

$$\sigma_{ij}^1(t) = \Omega_{\tau=\infty}^{\tau=t}{\varepsilon_{ij}^1(\tau)} \text{ in } V^1$$  \hspace{1cm} (8)

where in the case of the local scale, it is assumed that $\Omega_{\tau=\infty}^{\tau=t}$ is known a priori, for all constituents.

$$G_{ic}^l \geq G_{ic}^l \rightarrow \frac{\partial}{\partial \ell} (\partial V^1_\ell) > 0 \text{ in } V^1$$  \hspace{1cm} (9)

where $G_{ic}^l$ is the fracture energy release rate at the local scale and $G_{ic}$ is the energy release rate required for crack growth, which may be nonstationary for viscoelastic media (Knauss, 1970; Christensen, 1979; Costanzo and Allen, 1993). The index $i$ refers to the mode of fracture. Note that body forces have been neglected in Eq. 5.
Homogenization principles can now be used to establish the relationships connecting both length scales. Limiting to the case of small microstructures ($\ell^0 \gg \ell^1$) and (non-localized) statistically homogeneous distribution of microcracks, the following approximation can be used

$$\sigma_{ij}^0 = \frac{1}{V^0} \int_{V^0} \sigma_{ij}^1 dV \quad (10)$$

The use of Eq. 10 is termed a mean field theory because the higher order terms of the local scale stress field are dropped and the global analysis is performed only in terms of the mean stress.

Also, from the definition of mass density, it can be shown that

$$\rho^0 = \frac{1}{V^0} \int_{V^1} \rho^1 dV \quad (11)$$

Finally one needs to determine the constitutive relationship at the global length scale based on the constitution of the local scale. In multiscale analysis, however, there is no need to determine the global scale constitutive functional a priori, since it is determined concurrently as the analysis is performed. In the case of viscoelastic materials, the following incremental constitutive relations have been obtained by Souza and Allen (2009)

$$\Delta \sigma_{ij}^0 = C_{ijkl}^0(t) \Delta \varepsilon_{kl}^0 + \Delta \sigma_{R}^{R0} \quad (12)$$

$$C_{ijkl}^0 = \frac{1}{V^0} \int_{V^0} \left\{ C_{ijkl}^1 + C_{ijkl}^1 \left[ \left( \lambda_{ijkl}^1 \right)^2 + \lambda_{ijkl}^1 \right] \right\} dV \quad (13)$$

$$\Delta \sigma_{R}^{R0} = \frac{1}{V^1} \int_{V^1} \left( C_{ijkl}^{R1} \Delta \varepsilon_{kl}^{R1} + \Delta \sigma_{ij}^{R1} \right) dV \quad (14)$$

where $\lambda_{ijkl}^1$ is a localization quantity relating the local displacement field to the deformation on the external boundary of the RVE, $C_{ijkl}^0(t)$ is the homogenized instantaneous (tangent) constitutive tensor and $\Delta \sigma_{ij}^{R0}$ is the so-called homogenized history-dependent stress term, which represents the rate-dependence in the material (both bulk and cohesive zones) behavior and is recursively computed at each time step. Note that $C_{ijkl}^0(t)$ is a function of time through its dependence on the amount of damage accumulated at the local RVE, thus producing a nonlinear behavior at the global scale. A more detailed description of the multiscale model herein used can be found in Souza and Allen (2009). A key feature of this multiscale model is that it allows the computation of the full homogenized anisotropic tangent constitutive tensor by solving the local scale IBVP only once.

It is important to note that this multiscale model is said to be two-way coupled because the applied displacements on the boundary of the local scale (loading) are computed from the global scale strain tensor (global-to-local coupling) and the homogenized constitutive quantities, $C_{ijkl}^0$ and $\Delta \sigma_{ij}^{R0}$ (local-to-global coupling) are obtained from the solution of the local IBVP.

3. EXAMPLE PROBLEM - CYLINDER/PLATE IMPACT

Simple numerical simulations of a cylinder impacting a unidirectional carbon fiber reinforced composite plate are presented in this section. Simulations have been performed using the two-way coupled multiscale code based on the Finite Element Method presented in Souza and Allen (2009). All simulations have been run in a Dell workstation using 8 Intel Xeon processors at 3.00 GHz running under Linux Fedora 10. It is important to note that the amount of computational time is significantly reduced due to the use of parallel processors. In this case, different local scale meshes are solved in parallel by different processors.

The effects of different local microstructures, cracking and viscoelasticity are considered for this problem. Figure 2 shows a depiction of the problem, as well as its two-dimensional FE mesh for the global scale where the symmetry of the problem has been considered in order to reduce computational effort.

Two different microstructures are herein considered (see Figure 3) consisting of a unit cell and a trial RVE with five fibers. The volume fractions of fibers are 19.6% and 15.7%, respectively. The main purpose is to analyze the effect of different geometries on the overall mechanical behavior of the composite. The FE meshes have initially 390 and 1622 degrees of freedom for the unit cell and trial RVE, respectively. However, note that the number of degrees of freedom may increase as cohesive zone elements are inserted during the analyses.

Cohesive zone elements are herein automatically inserted in the local scale meshes as a initiation criterion is met; in this case once the traction exceeds a constant critical value. Stochastic descriptions of the critical traction for cohesive zone initiation can however significantly improve the modeling of real materials with defects. A Monte-Carlo model is currently under implementation where the critical tractions at different locations in the mesh follow the Weibull distribution.

Snapshots for the two cases where cracking is considered are given in Figure 4.
Figures 5 and 6 present the effect of the different microstructures on the response of the plate. The displacement component, $u_1^0$, at the center of the plate’s back face is shown in Figure 5, while the stress component, $\sigma_{22}^0$, at the same position is presented in Figure 6. It can be observed that the global response of the plate is affected by the microstructural geometric characteristics. This is one of the most important features of the multiscale approach, which can account for very important microstructural design variables such as particle volume fraction, relative position and orientation of microstructures.
particles, material constitution and fracture toughness of individual constituents, among others. Once again, stochastic models for the relative position and orientation of particles, and for the fracture toughness of individual constituents can drastically improve the use of this multiscale approach in practical application problems.

![Graph 1](image1.png)

**Figure 5:** Effect of microstructure: horizontal displacement at the center of the plate’s back face for the cases where cracking is not allowed.

![Graph 2](image2.png)

**Figure 6:** Effect of microstructure: 22-component stress at the center of the plate’s back face for the cases where cracking is not allowed.

The history of the back face 11-component stress, $\sigma_{11}^0$, is shown in Figure 7 for two different material constitutions (elastic and viscoelastic) and for the unit cell case where no cracking is allowed. For the elastic case, the Young’s modulus of the matrix material is assumed to be $E = E_0$, where $E_0$ is the corresponding value of the viscoelastic relaxation modulus when $t = 0$.

![Graph 3](image3.png)

**Figure 7:** Effect of material constitution: horizontal stress at the center of the plate’s back face for the unit cell case where cracking is not allowed.

Due to the short thickness dimension, rapid oscillations are observed in the elastic cases, which will last forever since there is no mechanism of energy dissipation. For the viscoelastic case however, rapid oscillations are observed initially.
at the same pace as the elastic oscillations (as shown in the close-up sub-figures), but they are damped out later due to viscoelastic energy dissipation.

4. CONCLUSIONS

This paper briefly described a two-way coupled multiscale model for predicting the performance of heterogeneous viscoelastic materials under impact. The evolution of microcracks in the material is modeled by the use of an algorithm to automatically insert cohesive zone elements in the local scale meshes. Material viscoelasticity is another source of energy dissipation accounted for in model. An example problem consisting of a plate/cylinder impact has been shown in order to demonstrate the model capabilities. Besides the fact that multiscale models can account for multiple sources of energy dissipation on the local scale, it has other important features in the design of composite (heterogeneous) materials: i) material characterization is needed only at the local scale; and ii) important design variables such as constituent volume fractions; particle spatial position and orientation; density, distribution, and orientation of defects; as well as other stochastic design variables, can be directly incorporated into the model.

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6. REFERENCES


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