INFLUENCE OF IMPERFECT BONDING ON INTERFACE OF MAGNETO-ELECTRO-ELASTIC HETEROSTRUCTURES: SH WAVES DISPERSION RELATIONS

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Abstract. In the present work the dispersion relations of stationary SH waves in a heterostructure with magneto-electro-elastic properties under imperfect mechanical bonding on the interface have been obtained. The calculations of the dispersion relations were based on the consideration of a symmetric system and results are presented by considering only the symmetric modes of the dispersion curves. Different limit cases are presented. The first dispersion branch for the symmetric modes is presented for five different values of a parameter used in the modeling of the imperfect bonding.

Keywords: Piezoelectricity, Piezomagnetism, Magneto-electro-elasticity, Dispersion curve, Wave propagation.

1. INTRODUCTION

The development of smart structures is currently receiving widespread attention owing to potential applications in several branches of engineering, such as in integrated control architecture with highly distributed sensors and actuators. More recently, applications can be found in the design of smart materials, where piezoelectric and piezomagnetic properties are involved. Here, the basic idea is to take advantage of each constituent’s properties and obtain a composite material that has superior magneto-electric coupling effects as compared to conventional piezoelectric or piezomagnetic materials.

The problem of wave propagation in this type of material has been studied by different authors using different geometries. Alshits et al. (1994) studied the existence of localized acoustic waves on the interface between two piezocrystals of arbitrary anisotropy. Aguiar and Angel (2000) investigated the scattering of antiplane waves from a slab region containing a random distribution of cylindrical cavities. Later, Aguiar and Angel (2006) obtained an integro-differential equation to describe the propagation of these waves inside the slab and showed that the second derivative of the corresponding displacement field has a square-root singularity near the boundary of the slab. Pan (2001) derived an exact closed-form solution for the static deformation of multilayered piezoelectric and piezomagnetic plates based on the quasi-Stroh formalism and the propagator matrix method. Pan and Heyliger (2002) extended the analytical method of Pan (2001) to the free vibration, three-dimensional, linear anisotropic, magneto-electro-elastic, simply supported, and multilayered rectangular plates. Wang et al. (2003) derived the state vector equations for three-dimensional, orthotropic, and linearly magneto-electro-elastic media. The solution of these equations is based on a mixed formulation, where the basic unknowns are not only displacements, electrical potential, and magnetic potential, but also stresses, electric displacements, and magnetic induction. Recently, Chen et al. (2007) presented an analytical treatment for the propagation of harmonic waves in infinitely extended, magneto-electro-elastic multilayered plates based on the state vector approach.

This work is motivated by recent contributions reported by Calas et al. (2008), Lombard and Piraux (2006) and Melkumyan and Mai (2008). The work of Melkumyan and Mai (2008) is used to obtain an extension of the results given by Calas et al. (2008) to the case of imperfect contact at the interface. Here, the behavior of stationary SH waves in a heterostructure with magneto-electro-elastic materials with imperfect mechanical bonding on the interface is studied. In particular, the case of permeable interfaces are discussed in detail. The governing system of partial differential equations is solved using the symmetry of the system and considering the superficial acoustic wave. Solutions can be separated into symmetric and anti-symmetric parts. The dispersion relations are derived only for the symmetric part of the solution. Different limit cases are presented. The dispersion curves and the influence of the imperfection at the interface are shown for some cases.

2. WAVE EQUATION FOR THE SH MODE

A material exhibiting magneto-electro-elastic properties with 6mm symmetry with magnetization and polarization in the $z$-axis direction is considered. It is well known (Calas et al., 2008) that the system of equations that governs the dynamic behavior of a homogeneous magneto-electro-elastic material has five coupled partial differential equations with three unknown components of the displacement filed, $u_x$, $u_y$, $u_z$, and two potential functions, $\phi$ and $\psi$.
Let us consider a plane wave propagating in an infinite homogeneous solid in a steady-state regime. In this case, the solution $U = (u_x, u_y, u_z, \varphi, \psi)^T$ of the system of equations is sought in the complex plane wave form, which is given by $U = U^0 \cdot e^{i(k \cdot \vec{r} - \omega t)}$, where $\vec{k}$ is the wave vector, $\omega$ is the circular frequency and $U^0 = (u_x^0, u_y^0, u_z^0, \varphi_0, \psi_0)^T$ is the amplitude vector. The wave front is in a plane that is perpendicular to the wave vector $\vec{k} = k_i \hat{i} + k_j \hat{j} + k_k \hat{k}$. Since both magnetic and electric effects appear in the SH mode, we confine the scope of the present work to them, and the equation describing the phenomena reduce to (Calas et al., 2008)

$$
\begin{align*}
\rho \frac{\partial^2 u}{\partial t^2} &= c^2 (\nabla^2 u + e \nabla^2 \varphi + \mu \nabla^2 \psi), \\
e \frac{\partial^2 \varphi}{\partial t^2} - g \nabla^2 \varphi &= 0, \\
f \frac{\partial^2 \psi}{\partial t^2} - \mu \nabla^2 \psi &= 0,
\end{align*}
$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $u = u_x$, $c = c_{44}$, $e = e_{15}$, $f = f_{15}$, $\mu = \mu_{15}$, $g = g_{11}$, and the material parameters $c, \varphi, \mu, g$ are the elastic, piezoelectric, piezomagnetic, dielectric, magnetic permeability, and magnetoelcetric coefficients, respectively.

The system of equations (1)-(3) describes the motion of a SH wave in an homogeneous material and, therefore, depends only on $(x, y, t)$. This is a coupled system of equations for the elastic displacement in the $z$-axis direction, $u$, the electric potential $\varphi$, and the magnetic potential $\psi$.

The system (1)-(3) can be solved using two auxiliary potential functions $\tilde{\varphi}$ and $\tilde{\psi}$ defined by

$$
\begin{align*}
\tilde{\varphi} &= \varphi - \frac{c}{e} u + \frac{g}{\mu} \psi, \\
\tilde{\psi} &= \psi - \frac{f}{\mu} u + \frac{e}{e} \varphi,
\end{align*}
$$

respectively. Replacing (4) into both (2) and (3), we obtain

$$
\begin{align*}
\nabla^2 \tilde{\varphi} &= 0, \\
\nabla^2 \tilde{\psi} &= 0.
\end{align*}
$$

Solving (4) for the functions $\varphi$ and $\psi$, we find

$$
\begin{align*}
\varphi &= \chi \tilde{\varphi} + \beta_1 u - \alpha_1 \tilde{\psi}, \\
\psi &= \chi \tilde{\psi} + \beta_2 u - \alpha_2 \tilde{\varphi},
\end{align*}
$$

where

$$
\chi = \left( \frac{e \mu}{e \mu - g} \right), \quad \alpha_1 = \left( \frac{\mu g}{e \mu - g} \right), \quad \alpha_2 = \left( \frac{e g}{e \mu - g} \right), \quad \beta_1 = \frac{e \mu - f g}{e \mu - g}, \quad \beta_2 = \frac{f e - e g}{e \mu - g}.
$$

Replacing the expressions given by Eq. (6) into Eq. (1), the typical wave motion equation for the mechanical displacement $u$ is obtained, being given by

$$
\left( \nabla^2 - \frac{1}{v_h^2} \frac{\partial^2}{\partial t^2} \right) u = 0,
$$

where $v_h$ is the wave propagation velocity in the medium and is given by

$$
v_h = \sqrt{c + e \beta_1 + f \beta_2}.
$$

Noting from above that all variables in this work are harmonic in time, we can drop the term $e^{i\omega t}$, and obtain the solution of Eq. (8) in the form

$$
u = (A^+ e^{-i k_y y} + A^- e^{i k_y y}) e^{-i \omega t},
$$

where $A^+$ and $A^-$ denote the amplitudes of the waves propagating to the right and to the left of the $y$-axis direction, respectively.

The solution of the system in Eq. (5) has the form

$$
\tilde{\varphi} = (A^+ e^{i k_y y} + A^- e^{-i k_y y}) e^{i \omega t}, \quad \tilde{\psi} = (A^+ e^{i k_y y} + A^- e^{-i k_y y}) e^{i \omega t},
$$

where $A^+$ and $A^-$ denote the amplitudes of the auxiliary electric potential related to the surface waves that are propagating in the positive and negative directions of the $y$-axis, respectively. On the other hand, $A^+$ and $A^-$ denote the amplitudes of the auxiliary magnetic potential related to the surface waves that are propagating in the positive and negative directions of the $y$-axis, respectively. The amplitudes are coupled through the electric and magnetic potentials.

Replacing the expressions of Eqs. (10) and (11) into Eq. (6), we finally get

$$
\varphi = \left[ \chi (A^+ e^{i k_y y} + A^- e^{-i k_y y}) + \beta_1 (A^+ e^{i k_y y} + A^- e^{-i k_y y}) - \alpha_1 (A^+ e^{i k_y y} + A^- e^{-i k_y y}) \right] e^{-i \omega t}.
$$

3. SH WAVES IN MAGNETO-ELECTRO-ELASTIC HETEROSTRUCTURE: DISPERSION RELATION

Let us now consider an infinite layer of width $d$ bonded imperfectly to two semi-infinite media, one at each side of the layer. The origin of the coordinate system is at the middle of the infinite layer, denoted medium II, such that the interfaces between the layer and the semi-infinite media, denoted media I and III, are located at $y = \pm d / 2$.

Since we are interested in the study of confined modes, the functions $u, \phi, \psi$ yield evanescent waves in media I and III. Considering that the semi-infinite media I and III are of the same material, these functions are symmetric with respect to the $xz$-plane and can be decoupled into symmetric and anti-symmetric modes. We then restrict the application of the boundary conditions on only one interface. Using Eqs. (10), (12), and (13), the functions $u, \phi, \psi$ can be written in the following form for the symmetric mode only.

$$u = e^{ixt}, \quad y \leq -d / 2$$
$$C' e^{i(\gamma + d/2)}, \quad y \geq d / 2$$

$$\beta' u + \chi' \Phi' e^{i(\gamma + d/2)} - \alpha' \Psi' e^{i(\gamma + d/2)}, \quad y \leq -d / 2$$

$$\phi = e^{ixt}, \quad \beta'' u + \chi'' \Phi'' \cosh(k_y y) - \alpha'' \Psi'' \cosh(k_y y), \quad y \leq |d / 2|,$$

$$\beta' u + \chi' \Phi' e^{-i(\gamma - d/2)} - \alpha' \Psi' e^{-i(\gamma - d/2)}, \quad y \geq d / 2$$

$$\psi = e^{ixt}, \quad \beta'' u - \alpha'' \Phi'' \cosh(k_y y) + \chi'' \Psi'' \cosh(k_y y), \quad y \leq |d / 2|.$$
Now, we refer to the mathematical statement of the boundary conditions at the interface between the layer and the semi-infinite media, which is located at $y = \pm d / 2$. In this case, we study the wave propagation with imperfect bonded effect, i.e., electromagnetically permeable bonding (Melkumyan and Mai, 2008), for which

\[ T^i = T^u = K(u^i - u^u), \quad \phi^i = \phi^u, \quad D^i = D^u, \quad \psi^i = \psi^u, \quad B^i = B^u, \]  

(22)

with six unknown amplitudes $\{A, C^i, \Phi^i, \Phi^u, \Psi^i, \Psi^u\}$ that appear in Eqs. (18)-(20). The first expression in Eq. (22) describes an elastic interface with elastic, or, spring constant $K > 0$. With this model, the interface is allowed to deform and the displacement at the interface can be discontinuous. The case $K \rightarrow \infty$ corresponds to a perfectly bonded interface and the case $K \rightarrow 0$ corresponds to mechanical free boundary with no elastic interaction at the interface.

Replacing Eqs. (18)-(20) into the boundary conditions given by Eq. (22), the dispersion relations for the symmetric modes can be written as

\[ K \left( \bar{\epsilon} \varepsilon k^i - \bar{\epsilon} \varepsilon k^u \tan (k^i d / \sqrt{2}) \right) + k^i \sinh (k^i d / \sqrt{2}) \left[ \left( \bar{\epsilon} \varepsilon - \bar{\epsilon} \varepsilon \frac{\bar{\epsilon} \varepsilon}{\varepsilon} \right) / N_2 + \left( \bar{\epsilon} \varepsilon - \bar{\epsilon} \varepsilon \frac{\bar{\epsilon} \varepsilon}{\varepsilon} \right) N_1 / N_2 \right] = 0, \]  

(23)

where

\[ N_1 = \left[ \beta^i \varepsilon^i + \beta^u \varepsilon^u \right] \coth (k^i d / 2) + \frac{\mu^i}{\varepsilon} \left[ \beta^i \varepsilon^i + \beta^u \varepsilon^u + \left( \beta^i \varepsilon^i - \beta^u \varepsilon^u \right) \varepsilon^i k^i \right], \]  

(24)

\[ N_2 = \left( \alpha N_1 / \beta^i + \beta^i \varepsilon^i \right) \sinh (k^i d / 2), \]  

(25)

and

\[ \bar{\beta}_i = (\beta^i - \bar{\beta}^u) K + \beta^u \varepsilon^i k^i, \quad \bar{\varepsilon} = \beta^u \varepsilon^i k^i, \quad \bar{\alpha} = -\left( \alpha^i \frac{\mu^i}{\varepsilon} + \alpha^u \varepsilon^i \coth (k^i d / 2) \right), \quad \bar{\bar{\varepsilon}} = \beta^u k^i \varepsilon^i \frac{\mu^i}{\varepsilon^i} + K \left( \chi^i \frac{\mu^i}{\varepsilon^i} + \chi^u \varepsilon^i \coth (k^i d / 2) \right). \]  

(26)

4. LIMIT CASES

The dispersion relations given by Eq. (23) satisfy different limit cases, i.e., the study is reduced to either perfect bonding $K \rightarrow \infty$ (permeable interface) or free boundary with no bonding $K \rightarrow 0$ (permeable interface).

a) Perfect bonding: $K \rightarrow \infty$.

\[ \left( \bar{\epsilon} \varepsilon k^i - \bar{\epsilon} \varepsilon k^u \tan (k^i d / \sqrt{2}) \right) + k^i \sinh (k^i d / \sqrt{2}) \left[ \left( \bar{\epsilon} \varepsilon - \bar{\epsilon} \varepsilon \frac{\bar{\epsilon} \varepsilon}{\varepsilon} \right) / N_2 + \left( \bar{\epsilon} \varepsilon - \bar{\epsilon} \varepsilon \frac{\bar{\epsilon} \varepsilon}{\varepsilon} \right) N_1 / N_2 \right] = 0, \]  

(27)

where $N_1$ and $N_2$ are given by Eqs. (24) and (25), respectively, with

\[ \bar{\beta}_i = (\beta^i - \bar{\beta}^u), \quad \bar{\varepsilon} = \beta^u k^i, \quad \bar{\alpha} = -\left( \alpha^i \frac{\mu^i}{\varepsilon} + \alpha^u \varepsilon^i \coth (k^i d / 2) \right), \quad \bar{\bar{\varepsilon}} = \beta^u k^i \varepsilon^i \frac{\mu^i}{\varepsilon^i} + K \left( \chi^i \frac{\mu^i}{\varepsilon^i} + \chi^u \varepsilon^i \coth (k^i d / 2) \right). \]

This case reproduces the same dispersion curves reported by Calas et al. (2008).

b) Free boundary with no bonding: $K \rightarrow 0$.

\[ \bar{\epsilon} \varepsilon k^i - \bar{\epsilon} \varepsilon k^u \tan (k^i d / \sqrt{2}) + k^i \sinh (k^i d / \sqrt{2}) \tan (k^i d / \sqrt{2}) \left[ \left( \bar{\epsilon} \varepsilon - \bar{\epsilon} \varepsilon \frac{\bar{\epsilon} \varepsilon}{\varepsilon} \right) / N_2 + \left( \bar{\epsilon} \varepsilon - \bar{\epsilon} \varepsilon \frac{\bar{\epsilon} \varepsilon}{\varepsilon} \right) N_1 / N_2 \right] = 0, \]  

(28)

where $N_1$ and $N_2$ are given by Eqs. (24) and (25), respectively, with

\[ \bar{\beta}_i = \beta^u \varepsilon^i k^i, \quad \bar{\varepsilon} = \beta^u \varepsilon^i k^i, \quad \bar{\alpha} = \beta^u k^i \varepsilon^i \frac{\mu^i}{\varepsilon^i}, \quad \bar{\bar{\varepsilon}} = \beta^u k^i \varepsilon^i \frac{\mu^i}{\varepsilon^i}. \]

5. NUMERICAL RESULTS

Numerical calculations were performed to show the quantitative and qualitative influence of bonding on the dynamic properties of layered composites. In the following numerical examples, we use the piezomagnetic material CoFe$_2$O$_4$ and the piezoelectric material PZT-4 to analyze the dispersion curves of the composite CoFe$_2$O$_4$/PZT-4/CoFe$_2$O$_4$. The
material parameters are available in Melkumyan and Mai (2008). In the study of SH-wave propagation in multi-layer systems, it is convenient to use the dispersion relation related to the phase velocity. The relation \( k_x = \omega / v_x \), where \( v_x \) is the phase velocity (Cheeke, 2002), is introduced into Eq. (17) and the components \( k_x^I \), \( k_x^II \) of the wave vectors in media I and II, respectively, are written as
\[
k_x^I = \omega \sqrt{\left(1/v_x^I\right)^2 - \left(1/v_x^I\right)^2}, \quad k_x^II = \omega \sqrt{\left(1/v_x^II\right)^2 - \left(1/v_x^I\right)^2}.
\]

The influence of the imperfect bonding at the interface of the composite medium can be observed in Figs. 1 and 2. Different values of the imperfection material parameter \( K \) are considered in order to show the influence of this parameter on the dispersion curves. Only the first dispersion branch is presented for five values of \( K \). The real parts of the solutions \( v_x \) are found as functions of the frequency \( f = \omega / 2\pi \). The thickness of the intermediate layer used in the calculations is 10 mm in Fig. 1 and 1 mm in Fig. 2. These figures suggest that the effect of the imperfection parameter \( K \) becomes more noticeable as the layer becomes thinner; especially, in the low frequency region.

Figure 1. Dispersion curves for the first mode considering five different values of \( K \) and \( d = 10 \text{ mm} \).

Figure 2. Dispersion curves for the first mode considering five different values of \( K \) and \( d = 1 \text{ mm} \).

Figure 3 shows the real part of the velocity with respect to the imperfection parameter \( K \) using a thickness layer of 1 mm and frequency of 0.051MHz. Notice that as \( K \) increases, the velocity increases asymptotically, approaching the velocity of perfect contact.
Fig. 3: Dependence of wave velocity on imperfect parameter $K$ in the CoFe$_2$O$_4$/PZT-4/CoFe$_2$O$_4$ composite.

6. CONCLUSIONS

In this paper, the dispersion relation for imperfect contact at the interface of a magneto-electro-elastic heterostructure considering superficial components in magnetic and electric potentials has been derived. Different limit cases ($K \to \infty$ and $K \to 0$) are studied for the general solution. The first dispersion branch is presented for five values of $K$. The magneto-electric coupling effect and the imperfect bonding have influences on the dispersion curves. In particular, the influence of $K$ becomes more noticeable as the layer becomes thinner.

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8. REFERENCES


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