CO-SIMULATION PROCEDURE FOR THE FINITE ELEMENT AND FLEXIBLE MULTIBODY DYNAMIC ANALYSIS

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Abstract. The structural flexibility of the mechanical components of the pantograph constitutes an issue seldom taken into account when evaluating the quality of the pantograph-catenary contact. A flexible multibody formulation in which the elastodynamics of each body is described with respect to a floating frame of reference is used. The mean axis conditions describe the reference conditions that ensure the uniqueness of the displacement field of each flexible body. In the applications foreseen for this methodology only linear elastic deformations of the bodies take place and, consequently, the deformation field of each flexible body can be described by a summation of deformation modes. Free-free vibration modes are used in all models presented in this work. Instead of involving the modal coordinates directly in the definition of any kinematic constraint, a virtual body is rigidly attached to the node, or nodes, of the flexible body in which a joint is to be defined. Then, the kinematic constraint equations are formulated using the virtual body, which for the purpose is defined as a rigid body with a null mass, instead of the flexible coordinates. The virtual bodies are also used to apply concentrated forces on the flexible body components. In this work several pantograph system models are built and demonstrated. The flexible pantograph models are used for the study of the contact of pantograph-catenary of an high-speed train moving at its nominal exploration velocity, of approximately 300 km/h. The catenary is modeled and simulated in a linear finite element code. By using a co-simulation procedure the dynamics of the pantograph, simulated in a multibody code, is effectively coupled with that of the catenary, described in a linear finite element code. The mean contact force, its standard deviation and the number of losses of contact serve as criteria to evaluate the contact quality.

Keywords: Integration time step, Flexible multibody dynamics, Contact mechanics, Pantograph-catenary contact

1. INTRODUCTION

The interaction between the pantograph and catenary is one of the factors that limits the operating speed of railway vehicles not only in the ability to collect energy at high operating speeds but also in the interoperability of the overhead equipment in trains and infrastructure. From the mechanical point of view the system must ideally run with relatively low contact forces, to minimize wear and damage of the contact elements, but with high enough contact forces to prevent contact loss, to ensure a constant power supply and minimize the occurrence of electric arching. Therefore, the study of the potential sources of perturbations of the pantograph and catenary performance, such as wind (Bocciolone et al., 2006; Pombo et al., 2009), extreme temperatures (EUROPAC, 2007) or flexibility effects, is of utmost importance. The flexibility of the pantograph is one of the aspects that can affect the quality of the energy collection that has not been approached up to now. The objective here is to study the pantograph using a flexible multibody approach.

Some of the earlier works in flexible multibody dynamics use fixed reference frames to describe the small elastic deformations given by the finite element method (Song and Haug, 1980) effectively coupling the rigid body motion and the small deformations. Shabana (1982, 1989) propose the use of sub-structuring and modal component synthesis to reduce the number of generalized coordinates that represent the flexible components, allowing for the analysis of complex shaped flexible multibody systems. The static correction modes, which represent the typical response of a structure subject to given boundary conditions, are normally used to complement the component mode synthesis (Yoo and Haug, 1986). Criteria to estimate the number of modes of vibration of each type has been proposed also (Wu and Haug, 1988). To enable the general use of different finite element types in the analysis of flexible multibody systems a lumped mass formulation, based on the diagonalization of the mass matrix that preserves the rotational inertia, is used here (Ambrósio and Gonçalves, 2001). The coupling terms, dependent of the type of finite elements used in the model and involving the derivation of the element shape functions, are not available in finite element literature (Chang and Shabana, 1990; Melzer, 1994). The diagonalization procedure prevents the need to derive these coupling elements, simplifying, in the process, the flexible multibody equations of motion.

The literature on the catenary-pantograph interaction emphasizes not only the mechanical aspects but also numerical simulation challenges due to the multi-physics characteristic of the problem. Based on a simple linear catenary models using 2D finite elements (Gardou, 1984), or on a catenary represented by cables and loaded by a lumped mass model of a pantograph (Jensen, 1997), several approaches have been proposed to keep the problem simple enough and tackle by a single code. It is claimed that the catenary structural deformations are basically linear and consequently the catenary systems are modeled using linear finite elements. The slacking of droppers is an exception being handled as a nonlinear
effect. Due to the multi-physics problem involved in the catenary-pantograph system, Veitl and Arnold (1999) proposed a co-simulation strategy between a code, where a catenary is described by the finite difference method and a commercial multibody code, used for the pantograph. A co-simulation approach, in which the finite element method is used to model in detail the catenary and a multibody dynamics approach is suited to handle the pantograph dynamics, using two separate codes is proposed in this work to handle the pantograph-catenary interaction dynamics.

2. FLEXIBLE MULTIBODY DYNAMICS BASICS FOR PANTOGRAPH MODELING

For the flexible body, shown in Fig. 1, let \( \mathbf{q}_i = [\mathbf{r}_i^T \mathbf{u}_i^T]^T \) be the vector of generalized coordinates of body \( i \), where \( \mathbf{r}_i = [\mathbf{r}_i^T \mathbf{p}_i^T]^T \) represents the translational and rotational position of body \( i \) local coordinate system \((\xi, \eta, \zeta)\), and vector \( \mathbf{u}_i \) represents body \( i \) elastic coordinates. The flexible body equations of motion are (Ambrósio and Gonçalves, 2001)

\[
\begin{bmatrix}
M_{rr} & M_{r\phi} & M_{rf} \\
M_{\phi r} & M_{\phi\phi} & M_{\phi f} \\
M_{fr} & M_{f\phi} & M_{ff}
\end{bmatrix}
\begin{bmatrix}
\ddot{r}_i \\
\ddot{\phi}_i \\
\ddot{u}_i
\end{bmatrix}
= 
\begin{bmatrix}
g_r \\
g_\phi \\
g_f
\end{bmatrix}
+ 
\begin{bmatrix}
s_r \\
s_\phi \\
s_f
\end{bmatrix}
- 
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & K_{i,ff}
\end{bmatrix}
\begin{bmatrix}
r_i^T \\
\phi_i^T \\
u_i^T
\end{bmatrix}
\] (1)

where the mass matrix \( M_i \) contains the mass, inertia tensor and inertia coupling terms, vector \( \mathbf{s}_i \) represents the velocity quadratic terms and other acceleration independent terms, \( \mathbf{g}_i \) is the generalized external force vector, and \( K_i \) is the finite elements stiffness matrix. The mass matrix in Eq. (1) may be either consistent or lumped. In order to maintain the inertia coupling terms independent of the finite element shape functions the lumped mass formulation is used in this work (Ambrósio and Gonçalves, 2001).

The equations of motion obtained do not have a unique displacement field. The gross motion of the flexible body coordinate system is represented by coordinates \( \mathbf{q}_i \), being necessary to impose a set of reference conditions to eliminate the rigid body modes and provide the unique displacement field of the flexible body. In general reference conditions are written as kinematic constraints that relate the independent and the dependent elastic coordinates. The mean axis conditions constraints are such that enforce the local frame \((\xi, \eta, \zeta)\) of body \( i \) to follow the motion of the nodes in such a way that the kinetic energy associated with the deformation corresponds to a minimum value for an observer stationary in the body local frame (Cavin and Dusto, 1977). The equations that define the mean axis reference conditions are

\[
\dot{\Phi}_{(ma)} = 
\begin{bmatrix}
\sum_{k=1}^{n} m_k \dot{\delta}_k \\
\sum_{k=1}^{n} m_k \ddot{\delta}_k + \sum_{k=1}^{n} \mu_k \dot{\theta}_k
\end{bmatrix} = 0
\] (2)

which may be written in more compact form as:

\[
\dot{\Phi}_{(ma)} = \Phi_{u}^{(ma)} \dot{u}' = 0
\] (3)

where \( \Phi_{u}^{(ma)} \) represents the Jacobian matrix of the mean axis reference conditions constraint equations. The time derivative of Eq. (3) results in the acceleration constraint equations of the mean axis reference conditions, written as

\[
\ddot{\Phi}_{(ma)} = \Phi_{u}^{(ma)} \ddot{u}' = \gamma_{(ma)}
\] (4)

The constraints associated to the mean axis conditions are imposed on the flexible body equations of motion, described by Eq. (1), leading to.
Note that the mean axis conditions are non-holonomic constraint conditions and can only be defined at the velocity and acceleration levels.

The flexible body equations of motion, shown in Eq. (5), include a large number of generalized coordinates, leading to an expensive computational procedure. For linear elastic small deformations, as those experienced by the pantograph components, it is possible to represent the deformation of the flexible body as a sum of deformation modes that are constant in time. The generalized elastic coordinates of body $i$ are described by a weighted sum of these modes as

$$u' = Xw$$

where $w$ represents the contributions of the modes of vibration towards the nodal displacements and $X$ the modal matrix containing a selected number of modes of vibration $\chi$ that are obtained by solving an eigenproblem. These modes of vibration correspond to those of a structure free in space. The vibration modes obtained, related to the first six lowest frequencies generally null, represent the rigid body motion of the flexible body that are removed from the modal matrix. By normalizing all modes of vibration $\chi$ with respect to the mass matrix $M$

$$X^T M_{ff} X = I$$

and

$$X^T K_{ff} X = \Lambda$$

The number of generalized elastic coordinates used in Eq. (8) is equal to the number of vibration modes included in the modal matrix, thus reducing considerably the problem dimension. The effects of local deformations induced by high concentrated loads originated, for example by kinematic constraint reaction forces or other force elements, can also be included in the modal synthesis using static correction modes (Yoo and Haug, 1986).

### 3. FINITE ELEMENT DYNAMICS BASICS FOR CATENARY MODELING

The motion of the catenary is characterized by small rotations and small deformations in which the only nonlinear effect is the slacking of the droppers and, therefore, they are typically modeled by using linear finite elements. The main catenary elements, the contact and messenger wires are modeled by using pre-tensioned Euler-Bernoulli beams. Using the finite element method, the equilibrium equations for the structural system are (Hughes, 1987)

$$Ma + Cv + Kx = f$$

where $M$, $C$ and $K$ are the finite element global mass, damping and stiffness matrices of the finite element model of the catenary, not to be confused with the finite element models of the flexible bodies used for the pantograph. The nodal displacements vector is $x$ while $v$ is the vector of nodal velocities, $a$ is the vector of nodal accelerations and $f$ is the vector with the applied forces. Equation (9) needs to be solved for $x$ or for $a$ depending on the integration method used to obtain the structural system dynamic response.

In this work the integration of the nodal accelerations uses a Newmark family integration algorithm (Newmark, 1957). The contact forces are evaluated for $t+\Delta t$ based on the position and velocity predictions for the FE mesh and on the pantograph predicted position and velocity. The finite element mesh accelerations are calculated by

$$\left(M + \gamma \Delta t C + \beta \Delta t^2 K\right)a_{t+\Delta t} = f_{t+\Delta t} - Cv_{t+\Delta t} - Kd_{t+\Delta t}$$
Predictions for new positions and velocities of the nodal coordinates of the linear finite element model of the catenary are found using the information of the last completed time step as

\[
\ddot{d}_{i+\Delta t} = \dot{d}_i + \Delta t \dot{v}_i + \frac{\Delta t^2}{2} (1 - 2\beta) a_i \quad \text{and} \quad \ddot{v}_{i+\Delta t} = \dot{v}_i + \Delta t (1 - \gamma) a_i. \tag{11}
\]

Then, with the acceleration \(a_{i+\Delta t}\), the positions and velocities of the finite elements at time \(t+\Delta t\) are corrected by

\[
\ddot{d}_{i+\Delta t} = \ddot{d}_{i+\Delta t} + \beta \Delta t^2 a_{i+\Delta t} \quad \text{and} \quad \ddot{v}_{i+\Delta t} = \ddot{v}_{i+\Delta t} + \gamma \Delta t a_{i+\Delta t}. \tag{12}
\]

This procedure is repeated until convergence is reached for a given time step.

4. CO-SIMULATION OF MULTIBODY AND FINITE ELEMENT CODES

The analysis of the pantograph-catenary interaction is done by two independent codes, one that uses a multibody formulation, for the pantograph, and another that uses a finite element software for the catenary, both being able to work as stand-alone. The structure of the communication between the codes is briefly described by Fig. 2. The multibody code provides the finite element code with the positions and velocities of the pantographs registration strips. The finite element code calculates the contact force, using the contact model represented in Fig. 2, and the location of the application points in the pantographs and catenary, using geometric interference. These forces are applied to the catenary, in the finite element code, and to the pantograph model, in the MB code. Each code handles separately the equations of motion of each sub-system based on the shared force information.

![Figure 2. Structure of the communication scheme between the MB and the FE codes](image)

The compatibility between the two integration algorithms imposes that the state variables of the two subsystems are readily available during the integration time but also that a reliable prediction of the contact forces is also available at any given time step. The key of the synchronization procedure between the multibody and finite element codes is the time integration, which must be such that it is ensured the correct dynamic analysis of the pantograph-catenary system, including the loss and regain of contact. The only restriction that is imposed in the integration algorithm of the multibody code is that its time step cannot exceed the time step of the finite element code.

5. APPLICATION TO THE STUDY OF A HIGH-SPEED PANTOGRAPH CATENARY SYSTEM

The models of the flexible pantographs are studied in a running scenario for which a single pantograph system is attached to the railway vehicle running at approximately 300 km/h on a straight track. The SNCF 25 kV catenary for high-speed tracks is represented in Fig. 3(a) while the CX pantograph for high-speed vehicles is depicted in Fig. 3(b).

![Figure 3. Overhead equipment for energy collection: (a) SNCF 25 kV catenary; (b) CX high-speed pantograph](image)
The pantograph flexible models considered in this work, shown in Fig. 4(a), are modeled using a rigid-flexible multibody approach. The two models in this work consider the upper arm flexibility and the flexibility of the upper and lower arms, respectively, allowing evaluating the influence of each one of the components on the pantograph dynamics and on the contact quality. Linear 3D beam elements are used to model the pantograph structures. The results depicted in Fig. 5 show that the dominant modes of vibration on the pantograph arms are the first bending modes. It is observed that only the modes of vibration associated to the three lower frequencies play a role in the dynamics of the arms.

![Figure 4](image.png)

**Figure 4.** Schematic representation of the pantograph models: (a) Flexible upper arm; (b) Flexible upper and lower arms

The bending of the upper arm results in the lowering the position of the contact points with the pantograph head, as depicted in Fig. 5. However, the differences observed on the contact kinematics are not reflected on the contact forces, which are similar for the rigid and flexible models as seen in Fig. 6.

![Figure 5](image.png)

**Figure 5.** Modal participation of the modes of vibration of the upper and lower flexible arms on their dynamic response

The bending of the upper arm results in the lowering the position of the contact points with the pantograph head, as depicted in Fig. 5. However, the differences observed on the contact kinematics are not reflected on the contact forces, which are similar for the rigid and flexible models as seen in Fig. 6.

![Figure 6](image.png)

**Figure 6.** Vertical position of the contact point for model with flexible: (a) upper arm; (b) upper and lower arms

![Figure 7](image.png)

**Figure 7.** Contact force on the pantograph-catenary interface; (a) Flexible upper arm; (b) flexible upper and lower arms
Although, for operational conditions and considering the present catenary model the influence of the deformation of the pantograph arms may be disregarded without loss of accuracy their effect under extreme conditions, or when excited close to its natural frequencies due to operational or defect conditions, should be taken into account.

6. CONCLUSIONS

The development of flexible multibody models of the pantograph is achieved in a straightforward way using the rigid multibody model as a base. The deformation of main frame of the pantograph that reacts to low frequency solicitations of the contact force does not influence the numerical results. The kinematic relations of the main-frame lead to a canceling effect of the deformation of the lower arm and of the top arm. It can be stated that for non-perturbed scenarios, for a pantograph running on a straight track at 300 km/h it is possible to disregard the effects due to the flexibility of the bodies. Nevertheless the use of flexible pantograph models is important to represent the dynamic response of the pantograph-catenary system to defects, which have the ability to excite its vibration modes.

7. ACKNOWLEDGEMENTS

The work presented has been partially developed within the European Project EUROPAC, contract STP4-CT-2005-012440. The collaboration of SNCF and Faiveley Transport in the work reported is specially acknowledged.

8. REFERENCES


9. RESPONSIBILITY NOTICE

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