EXPERIMENTAL STUDY AND MATHEMATICAL MODELING OF AIRFLOW IN GRAIN BULKS UNDER ANISOTROPIC CONDITIONS

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Abstract. A mathematical model, algorithm, and software were developed to calculate the static pressure, streamlines, and airflow velocity distribution in grain bulks for two- and three-dimensions under anisotropic conditions. The empirical relationships between permeability factors in horizontal and vertical directions (anisotropy factors) were analysed for soybean, wheat, maize, oats, rice, peas, flax seed and lentils mass. It was showed that anisotropy factor depends on the grain form and increases significantly with a deviation of this form from spherical. Anisotropy factor is rising with increase in air velocity, and the velocity influence rate varies from very weak for seeds with the form close to spherical (peas, soybean), up to essential, for grains with a greater deviation of the form from spherical (lentils, rice). In this work the relationship between the maximal area of grain contour projection on a horizontal plane (midsection for vertical flow) and the most probable value of the area of grain contour projection on a vertical plane (midsection for horizontal flow) was used as the principal parameter to specify the anisotropy factor of an anisotropy granular medium. As simulations show, there is difference between airflows through isotropic and anisotropic medium. This difference depends on grain type (value of anisotropy factor), area cross-section variation of storage bin (expansion ratio) and air inlet location. It was carried out numerical simulations of real and hypothetical aerated grain stores to detect the anisotropy influence on operational risk areas.

Keywords: Aeration, Anisotropy medium, 3-D airflow simulation, Aerated grain storage bins, Finite-element method

1. INTRODUCTION

Aeration (forced convection, ventilation) is widely used in grain stores to maintain and preserve the grain quality. Airflow distribution in grain bulk depends on various factors, including the filling method and grain bulk porosity, the depth of the grain, the grain morphology and configuration and size of the interstitial space in the mass, the air velocity, the form and size of any extraneous impurity in the mass, etc.

The influence of some of these parameters on airflow pattern in grain bulk has been studied by Shedd (1953), Brooker (1969), Bunn and Hukill (1963), Pierce and Thompson (1975), Haque et al. (1981), Jayas et al. (1987), Navarro and Noyes (2001), Khatchatourian and Savicki (2004), and Khatchatourian and Oliveira (2006). Gayathri and Jayas (2007) have presented a review of the reported mathematical models of airflow through grain mass. As works Hood and Thorpe (1992), Neethirajan et al. (2006) showed, there is a strong anisotropy of medium resistance to airflow through grain mass. For airflow in a silo where the area of cross-section as well as airflow direction practically does not vary, this effect can be neglected. However for the large grain storage bins with variable area the effect of anisotropy can be essential. Besides, the aeration of large grain stores is frequently carried out separately in different segments, thus, a vertical velocity component is not predominant in these conditions.

The principal objectives of the present work were:
(a) to create a mathematical model, algorithm, and software, to calculate the static pressure, streamlines, and airflow velocity distribution in two- and three-dimensions under anisotropic conditions;
(b) to study the relationship between the air velocity and the pressure gradient as a function of the pipe diameter, bed depth, filling method and airflow direction for wheat, oats, maize, rice and soybean mass; and
(c) to carry out numerical simulations of real and hypothetical grain stores with aeration to detect the anisotropy influence on operational risk areas.

2. MATHEMATICAL MODEL

Usually for problems of airflow through grain bulks, the equation of Navier-Stokes is replaced by empirical dependence $V = f(\text{grad}P)$, that represents nonlinear motion equation. In most of these equations the gradient of pressure is expressed as function of velocity by second-order parabola without a free term (Bear, 1988). The basic results of works for pressure-drop modelling in stored grain masses are compiled in ASAE (2000) and Navarro and Noyes (2001). In both these works it was used two-parameter model of Hukill and Ives (1955).
As work Khachatourian and Binelo (2008) showed, in large grain storage conditions it is more preferable to use three-parameter models, which permit to describe both the function $V = f(\text{grad}P)$ and its derivative. Their mathematical model, used in present study for simulation of the airflow for 2D and 3D cases, consists of a system:

$$\text{div} V = 0$$

$$V = -\frac{\text{grad}P}{|\text{grad}P|} \exp\left(\ln(1 + U^2) - 2U\arctan(U)\right) / \pi + 3U / 4a + c$$

where $U = a \ln|\text{grad}P| + b \pi$ is an intermediate argument; $a > 0, b, c$ are constants.

Eq. (2), which has replaced the Navier-Stokes equation, for 2D or 3D case can be written in the form:

$$V = K \cdot \text{grad}P$$

where $K$ is the second-rank hydraulic conductivity tensor for anisotropic medium.

If $x, y, z$ are principal directions (which are collinear to eigenvectors of matrix $K$), the tensor $K$ has diagonal form. Substituting Eq. (3) in Eq. (1), the nonlinear partial differential equation is obtained:

$$\frac{\partial}{\partial x} \left( -K_x \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( -K_y \frac{\partial P}{\partial y} \right) + \frac{\partial}{\partial z} \left( -K_z \frac{\partial P}{\partial z} \right) = 0$$

The boundary conditions for the problem considered have the form:

a) $P = P_0$, where $P_0$ is air entrance or exit pressure in Pa, and

b) $n \cdot K \cdot \text{grad}P = 0$, where $n$ is unit vector normal to the wall or floor surface.

Eq. (4) along with the boundary conditions describes the steady-state pressure and velocity distributions in a cross-section of aerated grain storage under non-uniform and anisotropic conditions. It is possible to admit, that $K_x = K_z \neq K_y$ and that coefficient of permeability $K_y$ is defined by the expression obtained of Eq. (2):

$$K_y = \exp\left(\ln(1 + U^2) - 2U\arctan(U)\right) / \pi + 3U / 4a + c$$

3. EXPERIMENTAL APPARATUS

To simulate the aerated grain storage characteristics, the equipment, showed in Fig. 1, was used. The experimental equipment consists of a centrifugal fan, an orifice-plate and a system of four small ‘silos’ composed of a polyvinyl chloride tubes (inside diameters from 0.05 m to 0.2 m and height of 1 m).

![Fig. 1 - Sketch of the experimental equipment](image-url)
resistance; it was used a cube with side = 0.4 m; 3) the relationship between airflow and pressure gradient for five kinds of seed (wheat, oats, maize, rice and soybean). In particular, it was analyzed the influence of pipe diameter and depth bed on resistance to airflow of grain mass. The obtained results allowed to find the minimal diameter of experimental camera (≤ 0.1 m) and the minimal depth bed (≤ 0.4 m) for analysis of resistance to airflow of grain mass, for which the experimental results don't depend on dimensions of the equipment. Therefore to obtain the empiric dependences the cameras of diameters of 0.1 m, 0.15 m and 0.2 m were used. The used depth beds were 0.6 m, 0.8 m and 1.0 m.

4. VALIDATION OF THE MATHEMATICAL MODEL FOR NON-HOMOGENEOUS ANISOTROPIC CONDITIONS IN A GRAIN BULK

The elements of the hydraulic conductivity tensor $K$ used in Eq. (4) have been determined of the relationship between velocity $V$ and pressure gradient $\frac{\partial P}{\partial y}$ along vertical and horizontal directions obtained experimentally. To estimate the anisotropy in this work it was used the anisotropy factor defined as the ratio of the effective (macroscopic) hydraulic conductivities along vertical and horizontal directions.

The relationships between air velocity $V$ and air pressure drop were obtained experimentally for vertical direction of airflow (Khatchatourian and Binelo, 2008). These data permit to calculate $K_y$ in Eq. (5).

If $x$, $y$, $z$ are principal directions the anisotropy factor can be estimated by ratio $\frac{K_x}{K_y} = \frac{\partial P}{\partial y} \frac{\partial P}{\partial x}$, which in this work was determined experimentally. Figure 2 presents the anisotropy factor variation with velocity for various grain types.

It is possible to see, that the grains having a greater eccentricity, form a medium with greater anisotropy factor. For example, this factor is almost equal to 1 for the peas which possess a nearly spherical form. On the contrary, the anisotropy factor values for lentils and oats are much greater.

As experimental data show, the dependence between $\ln(K/K_y)$ and $\ln(V)$ can be considered as linear ($R^2>0.943$), i.e. can be presented in the form:

$$\ln\left(\frac{K_x}{K_y}\right) = s \ln V + d,$$  \hfill (6)

where $s$ is slope and $d$ is constant term.

At filling, the grains lay down so that the potential energy of system would be the least (the barycentre of each grain tends to occupy the lowest position). It is possible to guess, that value of anisotropy factor for the considered medium is related to the ratio of midsections of grain in vertical and horizontal directions. Let $A_{\text{min}}$, $A_{\text{medium}}$, and $A_{\text{max}}$ be the areas of projections of grain contours on its orthogonal sections respective to principal inertia moments. In this work it is admitted, that both greatest and medium axes of inertia are located in a horizontal plane and midsection of every grain for vertical direction of airflow is constant ($A_{\text{max}}$). The greatest axis of grain can occupy the any direction in this plane and midsection of grain for horizontal airflow varies between $A_{\text{medium}}$ and $A_{\text{min}}$.

To generalize the experimental data for anisotropy factor variation the different methods for determination of grain midsection were compared. In particular, it was used the area of grain contour projection: a) in direction of the greatest axis ($A_{\text{max}}$), b) in direction of the intermediate axis ($A_{\text{medium}}$), c) their arithmetic average ($A_{\text{min}}+A_{\text{medium}})/2$, and, d) their geometrical mean $\sqrt{A_{\text{min}} \cdot A_{\text{medium}}}$. But the best generalization has been obtained at use of most probable estimate area $A_{\text{mpe}} = A_{\text{min}} + 2(A_{\text{medium}} - A_{\text{min}})/\pi$, where $\varphi \in [0,\pi/2]$ is an angle of deflection of the greatest axis of grain from a plane orthogonal to a horizontal airflow.

Digital image processing was used to obtain the morphological seed characteristics. Figure 3 shows the relations between $A_{\text{max}}$ and ($A_{\text{min}}+A_{\text{medium}})/2$, $\sqrt{A_{\text{min}} \cdot A_{\text{medium}}}$ and $A_{\text{mpe}}$ for airflow through the rice mass.

The fitted curves for variation of slope $s$ and constant term $d$ of Eq. (6) with variation of ratio $A_{\text{max}}/A_{\text{mpe}}$ obtained by the least square (LS) method were presented in exponential and linear form. Applying these dependences and Eq. (6) for various grain kinds the predicted curves were simulated (continuous lines in Fig. 2).

5. SOFTWARE DESCRIPTION AND DEVELOPMENT

The nonlinear partial differential equation for static pressure Eq. (4) was solved by the finite element method. The software was developed in ANSI C++ and Dev-Pascal and consisted of tools for geometry construction and mesh generation, generation of system matrix by the finite element method and solver of obtained system of linear algebraic equations and tool for results three-dimensional presentation and analysis. It was used free-of-charge software when possible. To generate mesh it was used Tetgen, available under GPL license. To resolve the problem the software executes four iterative processes: 1) it calculates the hydraulic conductivity tensor for anisotropic medium in each point of the integration domain; 2) it resolves the system of linear algebraic equations by the successive over-relaxation
(SOR) method; 3) it searches the system design point, located in the performance curve of the aerator fan; and 4) it adaptively refines the mesh. An output file is generated in VTK (Visualization toolkit, http://www.vtk.org/) format. To visualise the results Paraview software (http://www.paraview.org/), available free of charge was used.

![Graph showing Anisotropy factor, $K_x/K_y$ vs Velocity, m s$^{-1}$](image)

**Fig. 2** – Observed and predicted relationships between horizontal and vertical permeability for various seed kind; ■, lentils; ●, flax seed; ▲, white oats; □, rice; ▼, black oats; ○, maize; Δ, wheat; ◦, soybean; •, peas; closed points: Alagusundaram et al. (1992); open points: Authors; −, predicted (Authors).

![Graph showing Areas ratio vs Grain number](image)

**Fig. 3** - Relations between midsection in direction vertical ($A_{max}$) and examined areas in horizontal plane for rice; ■, $A_{max}/A_{min}$; ●, $A_{max}/A_{medium}$; ○, $A_{max}/(A_{medium}+A_{min})/2$; ▲, $A_{max}/\sqrt{A_{min} \cdot A_{medium}}$; ▼, $A_{max}/(A_{min}+2(A_{medium}-A_{min})/\pi)$.  

6. **NUMERICAL SIMULATIONS**

The influence of anisotropy factor on the airflow through grain bulk and on the aeration system efficiency was analyzed for storage bins with three air entrances (Fig. 4). The aeration was carried out: a) simultaneously through all entrances; b) separately through one of three entrances. It was used the global airflow rate of $Q=9$ m$^3$ h$^{-1}$ t$^{-1}$ (2.5 $\times$ 10$^{-6}$ m$^3$ s$^{-1}$ kg$^{-1}$), which is the most commonly recommended value for aerated grain storage. To estimate the efficiency of an aeration system in this work it was used the local specific airflow rate $q_L$ introduced in Khatchaturian and Binelo (2008). Figure 5 presents the comparison of airflow in central cross-section of storage bins with different aeration systems for anisotropic ($A_{max}/A_{npe}=1.6$) and isotropic medium. Four air inlet systems were studied: (a) upper lateral, lower lateral and central inlets; (b) upper lateral inlet; (c) lower lateral inlet; and (d) central inlet.

As simulations show, there is difference between airflows through isotropic (grey lines) and anisotropic (black lines) medium for all considered cases and this difference increases for grain bulks with higher ratio $A_{max}/A_{npe}$. Due to larger easiness to move in the horizontal direction in anisotropic medium the air movement increases in this direction. Therefore for airflow through upper lateral inlet (b) and central inlet (d) this difference is more than for airflow through lower lateral inlet (c) or for case of aeration simultaneously through all inlets (a).
Fig. 4 - Outline sketch of simulated store bin.

To estimate the anisotropy effect on the risk domains in the grain storage bin, the distribution of local specific airflow rates was studied for separated operation of the upper lateral (one time period), lower lateral (two time periods) and central inlets (three time periods). In Fig. 6 the black lines represent contour curves generated by points with constant values of local airflow specific rates calculated for anisotropic medium. For isotropic medium these curves are represented by white lines. The values of local specific airflow rate $q_L$ for anisotropic case are more uniform for deep layers. The zone of limited aeration ($q_L < 3$), located in top part of storage bin, increases for anisotropy case, because the part of the air that moved along horizontal direction is more for anisotropic medium than for isotropic medium.

Thus the simulations show, there is difference between airflows through isotropic and anisotropic medium. This difference depends on grain type (value of anisotropy factor), area cross-section variation of storage bin (expansion ratio) and air inlet location. If expansion ratio=1 (for silo case), there is no difference in airflow for isotropic and anisotropic cases.

Fig. 5 - Anisotropy influence on airflow in storage bins with different aeration systems; anisotropic medium $A_{max}/A_{mp}=1.6$ (maize): black lines; isotropic medium: grey lines.

7. CONCLUSIONS

The ratio of permeability factors in horizontal and vertical directions depends on the grain form and increases significantly with a deviation of this form from spherical. This ratio is rising with the increase in air velocity, and the velocity influence rate varies from very weak for seeds with the form close to spherical (peas, soybean), up to essential, for grains with a greater deviation of the form from spherical (lentils, rice). It was showed that the relationship between the maximal area of a grain contour projection on a horizontal plane (midsection for vertical flow) and the most probable value of the area of a grain contour projection on a vertical plane (midsection for horizontal flow) can be used as the principal parameter to specify the anisotropy factor of an anisotropy medium.

As simulations show, there is difference between airflows through isotropic and anisotropic medium. This difference depends on grain type (value of anisotropy factor), area cross-section variation of storage bin (expansion ratio) and air inlet location.

The distribution of local specific airflow rates $q_L$ for anisotropic case is more uniform for deep layers of aerated storage bin. The zone of limited aeration ($q_L < 3$), located in top part of storage bin, increases for anisotropy case, because the part of the air that moved along horizontal direction is more for anisotropic medium than for isotropic medium.
Fig. 6 - Distribution of resultant local specific airflow rates for isotropic (white lines) and anisotropic (black lines) grain bulks with separated functioning inlets, $Q = 9 m^3 h^{-1} r^{-1} (2.5 \times 10^{-6} m^3 s^{-1} kg^{-1})$; $A_{max}/A_{mpe}=1.6$ (maize).

8. REFERENCES

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