INTERNAL LOADING DISTRIBUTION IN STATICALLY LOADED BALL BEARINGS SUBJECTED TO A COMBINED RADIAL, THRUST, AND MOMENT LOAD, INCLUDING THE EFFECTS OF TEMPERATURE AND FIT

Mário César Ricci, mariocesarricci@uol.com.br
Brazilian Institute for Space Research/Space Mechanics and Control Division (INPE/DMC)

Abstract. A new, rapidly convergent, numerical procedure for internal loading distribution computation in statically loaded, single-row, angular-contact ball bearings, subjected to a known combined radial, thrust, and moment load, is used to find the load distribution differences between a loaded unfitted bearing at room temperature, and the same loaded bearing with interference fits which might experience radial temperature gradients between inner and outer rings. For each step of the procedure it is required the iterative solution of $Z + 3$ simultaneous nonlinear equations – where $Z$ is the number of the balls – to yield exact solution for axial, radial, and angular deflections, and contact angles. Numerical results are shown for a 218 angular-contact ball bearing.

Keywords: ball, bearing, static, load, temperature, fit

1. INTRODUCTION

Ball and roller bearings, generically called rolling bearings, are commonly used machine elements. They are employed to permit rotary motions of, or about, shafts in simple commercial devices and also used in complex engineering mechanisms.

This work is devoted to study of the internal loading distribution in statically loaded ball bearings. Several researchers have studied the subject as, for example, Strieber (1907), Sjövall (1933), Jones (1946) and Rumbarger (1962). The methods developed by them to calculate distribution of load among the balls and rollers of rolling bearings can be used in most bearing applications because rotational speeds are usually slow to moderate. Under these speed conditions, the effects of rolling element centrifugal forces and gyroscopic moments are negligible. At high speeds of rotation these body forces become significant, tending to alter contact angles and clearance. Thus, they can affect the static load distribution to a great extent.

Harris (2001) described methods for internal loading distribution in statically loaded bearings addressing pure radial; pure thrust (centric and eccentric loads); combined radial and thrust load, which uses radial and thrust integrals introduced by Sjövall; and for ball bearings under combined radial, thrust, and moment load, initially due to Jones.

There are many works describing the parameters variation models under static loads but few demonstrate such variations in practice, even under simple static loadings. The author believes that the lack of practical examples is mainly due to the inherent difficulties of the numerical procedures that, in general, deal with the resolution of various non-linear algebraic equations that must to be solved simultaneously.

In an attempt to cover this gap studies are being developed in parallel (Ricci, 2009 to Ricci, 2009e). Particularly in this work a new, precise numerical procedure, described in Ricci (2009c), for internal load distribution computation in statically loaded, single-row, angular-contact ball bearings subjected to a known external combined radial, thrust, and moment load, is used to find the load distribution differences between a loaded bearing with clearance fits at room temperature, and the same loaded bearing with interference fits, such might experience radial temperature gradients between inner and outer rings.

In the most usual situation, angular contact bearings would first be fitted, with interference or clearance defined at room temperature, to their respective shaft and housing; then a defined axial “hard” preload would be applied and subsequently in operation the bearings might experience radial temperature gradients between inner and outer rings.

Ball bearings and other radial rolling bearings are designed to have a diametral clearance. Due to this radial clearance the bearing also can experience an axial play. Removal the axial freedom causes the ball-raceway contact line to assume an oblique angle with respect to the radial plane; hence, a contact angle different from zero will occur. This angle is called free contact angle and is a function of radial clearance and the raceway groove curvatures.

Press or shrink fitting of the inner ring on the shaft causes the inner ring to expand slightly. Similarly, press fitting of the outer ring in the housing causes the former member to shrink slightly. Thus, the bearing’s diametral clearance will tend to decrease. Large amounts of interference in fitting practice can cause bearing clearance to vanish and even produce negative clearance or interference in the bearing.

Thermal conditions of bearing operation can also affect the diametral clearance. Heat generated by friction causes internal temperatures to rise. This in turn causes expansion of the shaft, housing, and bearing components. Depending on the shaft and housing materials and on the magnitude of thermal gradients across the bearing and these supporting structures, clearance can tend to increase or decrease.
2. STATIC LOAD DISTRIBUTION UNDER COMBINED RADIAL, THRUST, AND MOMENT LOAD IN BALL BEARINGS

Having defined in other works analytical expressions for geometry of bearings and the contact stress and deformations for a given ball or roller-raceway contact (point or line loading) in terms of load (see, e.g., Harris, 2001) it is possible to consider how the bearing load is distributed among the rolling elements. In this section a specific load distribution consisting of a combined radial, thrust, and moment load, which must be applied to the inner ring of a statically loaded ball bearing, is given.

According Ricci (2009c), let a ball bearing with a number of balls, \( Z \), symmetrically distributed about a pitch circle according to Fig. 1a, to be subjected to a combined radial, thrust, and moment load. Then, a relative axial displacement, \( \delta_a \), a relative angular displacement, \( \theta \), and a relative radial displacement, \( \delta_r \), between the inner and outer ring raceways may be expected. Let \( \psi = 0 \) to be the angular position of the maximum loaded ball.

Figure 1a shows the ball angular positions in the radial plane that is perpendicular to the bearing’s axis of rotation, \( \Delta \psi = 2\pi/Z \), \( \psi_j = 2\pi/Z(j-1), j = 1…Z \); (b) Initial and final curvature centers positions at angular position \( \psi \), with and without applied load.

Figure 1b shows the initial and final groove curvature centers positions at angular position \( \psi \), before and after loading, considering the centers of curvature of the raceway grooves fixed with respect to the corresponding raceway. If \( \delta_a, \theta, \) and \( \delta_r \) are known, the contact angle at angular position \( \psi \), after the combined load has been applied, is given by

\[
\beta(\psi) = \cos^{-1}\left( \frac{A \cos \beta_f + \delta_r \cos \psi}{A + \delta_n} \right),
\]

where \( A \) is the distance between raceway groove curvature centers, \( \beta_f \) is the free contact angle, and \( \delta_n \) is the total ball normal deflection.

Also,

\[
\delta_a + R_i \theta \cos \psi = (A + \delta_n) \sin \beta - A \sin \beta_f,
\]

where

\[
R_i = d_e/2 + (f_i - 0.5)D \cos \beta_f
\]

expresses the locus of the centers of the inner ring raceway groove curvature radii, with \( d_e \) being the bearing pitch diameter, \( D \) the ball diameter, and \( f_i \) the inner race conformity ratio.

From Fig. 1b we can arrive in the transcendental equations for the extend of the loading zone, \( \psi_l \), that are

\[
\cos \beta_f - \cos \beta_l = \frac{\delta_r}{A} \cos \psi_l
\]
and
\[ \sin \beta_i - \sin \beta_j = \frac{\delta_i}{A} + \frac{R \theta}{A} \cos \psi, \]  
(4)
where \( \beta_i \) is the contact angle regarding the end of loading zone.

From Eq. 1, the total normal approach between two raceways at angular position \( \psi \), after the combined load has been applied, can be written as
\[ \delta_n(\psi) = A \left( \frac{\cos \beta_j}{\cos \beta} - 1 \right) + \frac{\delta_j \cos \psi}{\cos \beta}. \]  
(5)

From Fig. 1b and Eq. 5 it can be determined that \( s \), the distance between the centers of the curvature of the inner and outer ring raceway grooves at any rolling element position \( \psi \), is given by
\[ s(\psi) = A + \delta_n = A \frac{\cos \beta_j}{\cos \beta} + \frac{\delta_j \cos \psi}{\cos \beta}. \]  
(6)

From Eqs. 2 and 6 yields, for \( \psi = \psi_j \),
\[ \delta_a + R \theta \cos \psi_j - \delta_j \tan \beta_j \cos \psi_j - A \frac{\sin \left( \beta_j - \beta_f \right)}{\cos \beta_j} = 0, \quad j = 1, \ldots, Z. \]  
(7)

From load-deflection relationship for ball bearings and Eq. 5 yields, for \( \psi = \psi_n \),
\[ Q_j = K_n \left[ A \left( \frac{\cos \beta_f}{\cos \beta_j} - 1 \right) + \frac{\delta_j \cos \psi_j}{\cos \beta_j} \right]^{3/2}, \quad j = 1, \ldots, Z. \]  
(8)

If a thrust load, \( F_a \), a radial load, \( F_r \), and a moment, \( M \), are applied then, for static equilibrium to exist
\[ F_a = \sum_{j=1}^{Z} Q_j \sin \psi_j, \]  
(9)
\[ F_r = \sum_{j=1}^{Z} Q_j \cos \beta_j \cos \psi_j, \]  
(10)
and
\[ M = \sum_{j=1}^{Z} Q_j \sin \beta_j \left[ \left( R_i + \delta_j \cos \psi_j \right) \cos \psi_j - \delta_j \right]. \]  
(11)

Substitution of Eq. 8 into Eq. 9 yields
\[ F_a - \sum_{j=1}^{Z} K_n \sin \beta_j \left[ A \left( \frac{\cos \beta_f}{\cos \beta_j} - 1 \right) + \frac{\delta_j \cos \psi_j}{\cos \beta_j} \right]^{3/2} = 0. \]  
(12)

Similarly,
\[ F_r - \sum_{j=1}^{Z} K_n \cos \psi_j \cos \beta_j \left[ A \left( \frac{\cos \beta_f}{\cos \beta_j} - 1 \right) + \frac{\delta_j \cos \psi_j}{\cos \beta_j} \right]^{3/2} = 0, \]  
(13)
and
\[ M - \sum_{j=1}^{Z} K_n \left[ A \left( \frac{\cos \beta_f}{\cos \beta_j} - 1 \right) + \frac{\delta_j \cos \psi_j}{\cos \beta_j} \right]^{3/2} \sin \beta_j \left[ \left( R_i + \delta_j \cos \psi_j \right) \cos \psi_j - \delta_j \right] = 0. \]  
(14)
3. EFFECTS OF INTERFERENCE FITTING, THERMAL GRADIENTS, AND SURFACE FINISH ON CLEARANCE

In this section, the principal relationships between interference fittings, thermal gradients, surface finish and changes in diametral clearance are summarized. As described in Harris (2001), the increase in the inner raceway diameter, \( d_o \), due a press fitting between a bearing inner ring and a shaft of hole diameter \( d_s \), is given by

\[
\Delta_\alpha = \left[ \frac{2I d_o / d_h}{(d_o / d_s)^{3/2} - 1} \right] \left[ \frac{(d_o / d_s)^{3/2} + 1 + \nu_b}{E_b \left[ (d_o / d_s)^{3/2} - 1 \right]} \right] \, \text{mm}
\]

where \( I, d_o, E_b, \nu_o, \nu_b \) and \( \nu_b \) are diametral interference, bearing inner diameter, modulus of elasticity for inner ring and shaft, respectively.

If the bearing inner ring and shaft are both fabricated from the same material, then

\[
\Delta_\alpha = I d_o / d_s \, \text{mm}
\]

Similarly, the decrease in the outer raceway diameter, \( d_a \), due a press fitting between a bearing outer ring and a housing of outside diameter \( d_i \), is given by

\[
\Delta_\beta = \left[ \frac{2I d_o / d_h}{(d_o / d_s)^{3/2} - 1} \right] \left[ \frac{(d_o / d_s)^{3/2} + 1 + \nu_b}{E_b \left[ (d_o / d_s)^{3/2} - 1 \right]} \right] \, \text{mm}
\]

where \( d_s \) is the bearing outer diameter.

If the bearing outer ring and housing are both fabricated from the same material, then

\[
\Delta_\beta = I d_o / d_i \, \text{mm}
\]

A reduction in \( I \) due to surface finish must be taking into account (Harris, 2001).

Now, considering bearing outer and inner rings at temperatures \( T_o - T_a \) and \( T_i - T_a \) above ambient, respectively, the approximate increases in \( d_o \) and \( d_i \) are \( \Gamma_o d_o(T_o - T_a) \) and \( \Gamma_i d_i(T_i - T_a) \), respectively. Thus the diametral clearance increase due to thermal expansion is

\[
\Delta_\epsilon = \Gamma_o d_o(T_o - T_a) + \Gamma_i d_i(T_i - T_a)
\]

When the housing and shaft are not fabricated from the same material (usually steel) as the bearing, an increase in the interference can be wait, that are given by \( (\Gamma_b - \Gamma_a) d_a(T_o - T_a) \) and \( (\Gamma_b - \Gamma_a) d_b(T_i - T_a) \), respectively.

Considering a bearing having a clearance \( P_o \) prior to mounting at room temperature, the change in clearance, after mounting with bearing outer and inner rings at temperatures \( T_o \) and \( T_i \), respectively, is given by

\[
\Delta P_o = \Delta_{\epsilon} - \Delta_\alpha - \Delta_\beta
\]
4. NUMERICAL RESULTS

To show an application of the theory developed in this work a numerical example is presented, which uses the Newton-Rhapson method to solve the simultaneous nonlinear equations 7, 12, 13 and 14.

I have chosen the 218 angular-contact ball bearing as example, which was also used by Harris. The 218 angular-contact ball bearing has a 0.09 m bore, a 0.16 m o.d. and is manufactured to ABEC 7 tolerance limits. The bearing is mounted on a hollow steel shaft of 0.0635 m bore with a k6 fit and in a titanium housing having a effective o.d. of 0.2032 m with an M6 fit. Considers that the inner ring operates at a mean temperature of 148.9°C, that the outer ring is mounted on a hollow steel shaft of 0.0635 m bore with a k6 fit and in a titanium housing having a effective o.d. of 0.2032 m with an M6 fit. Considers that the inner ring operates at a mean temperature of 148.9°C, that the outer ring is mounted on a hollow steel shaft of 0.0635 m bore with a k6 fit and in a titanium housing having a effective o.d. of 0.2032 m with an M6 fit. Considers that the inner ring operates at a mean temperature of 148.9°C, that the outer ring is mounted on a hollow steel shaft of 0.0635 m bore with a k6 fit and in a titanium housing having a effective o.d. of 0.2032 m with an M6 fit.

There are three steps in the numerical procedure. The first, considering the bearing unfitted at assembling temperature; the second, considering the fits above at assembling temperature; and the third, considering the fits above at operational temperatures for the inner and outer rings. Before each step the geometry of the bearing is obtained from which, the nonlinear equations are solved simultaneously to obtain radial and axial deflections and contact angles.

Figures 2 to 4 show some parameters, as functions of the applied thrust load, for the three steps of the procedure, for some values of the applied radial load and for moment values of 0 Nm and 100 Nm.

Figures 2a and 2b show the normal ball loads for the maximum loaded ball and for the loaded ball located at \( \psi = 180^\circ \), respectively. For higher applied radial loads and for the range of thrust and moment loads adopted, the normal ball load of the maximum loaded ball is nearly constant. The normal ball load of the loaded ball located at \( \psi = 180^\circ \) increase monotonically with the thrust load for all values of the applied radial and moment loads considered.

Figures 3a and 3b show the contact angle for the maximum loaded ball and for the loaded ball located at \( \psi = 180^\circ \), respectively. The straight lines represent the free contact angles for the three steps of the procedure. For higher applied radial loads and for the range of thrust and moment loads adopted, the contact angle of the maximum loaded ball (of the ball located at \( \psi = 180^\circ \)) is always less than (greater than) the free contact angles.

Figures 4a and 4b show the axial and radial deflections, respectively.
5. CONCLUSIONS

The importance of this work lies in the fact that it uses a new procedure for get numerically, accurately and quickly, the static load distribution of a ball bearing under radial, thrust, and moment loading, taking into account the influence of fits and thermal gradients. Precise applications, as for example, space applications, require a precise determination of the static loading. Models available in literature are approximate and often are not compatible with the desired degree of accuracy. This work can be extended to determine the loading on high-speed bearings where centrifugal and gyroscopic forces do not be discarded. The results of this work can be used in the accurate determination of the friction torque of the ball bearings, under any operating condition of temperature and speed.

6. REFERENCES

Ricci, M. C., 2009d. “Internal loading distribution in statically loaded ball bearings subjected to a combined radial, thrust, and moment load”, Proceedings of the 60th International Astronautical Congress, October, 12-16, Daejeon, South Korea. ISSN 1995-6258.
Ricci, M. C., 2009e. “Internal loading distribution in statically loaded ball bearings subjected to an eccentric thrust load”, accepted to publication in Mathematical Problems in Engineering.

7. RESPONSIBILITY NOTICE

The author is the only responsible for the printed material included in this paper.