



UNIVERSIDADE DE SÃO PAULO
ESCOLA DE ENGENHARIA DE SÃO CARLOS
Departamento de Engenharia de Estruturas



TABELAS DE VIGAS: Deslocamentos e Momentos de Engastamento Perfeito

Revisão e adaptação:

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Colaboração:

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São Carlos, fevereiro de 2010

TABELA 3.1a				
DESLOCAMENTOS ELÁSTICOS EM VIGAS				
CASO	VINCULAÇÃO E CARREGAMENTO	FLECHA		EQUAÇÃO DA ELÁSTICA
		w_{max}	x	
1		$\frac{1}{8} \frac{p\ell^4}{EI}$	0	$\frac{p\ell^4}{24EI} (\alpha^4 - 4\alpha + 3)$
2		$\frac{1}{30} \frac{p\ell^4}{EI}$	0	$\frac{p\ell^4}{120EI} (\alpha^5 - 5\alpha + 4)$
3		$\frac{11}{120} \frac{p\ell^4}{EI}$	0	$\frac{p\ell^4}{120EI} (-\alpha^5 + 5\alpha^4 - 15\alpha + 11)$
4		$\frac{1}{3} \frac{P\ell^3}{EI}$	0	$\frac{P\ell^3}{6EI} (\alpha^3 - 3\alpha + 2)$
5		$\frac{1}{2} \frac{M\ell^2}{EI}$	0	$\frac{M\ell^2}{2EI} (1 - \alpha)^2$
6		$\frac{5}{384} \frac{p\ell^4}{EI}$	0,5l	$\frac{p\ell^4\alpha}{24EI} (\alpha^3 - 2\alpha^2 + 1)$
7		$\frac{3}{460} \frac{p\ell^4}{EI}$ (*)	0,519l	$\frac{p\ell^4\alpha}{360EI} (3\alpha^4 - 10\alpha^2 + 7)$
8		$\frac{1}{120} \frac{p\ell^4}{EI}$	0,5l	$\frac{p\ell^4\alpha}{960EI} (16\alpha^4 - 40\alpha^2 + 25)$ (**)
9		$\frac{1}{48} \frac{P\ell^3}{EI}$	0,5l	$\frac{P\ell^3\alpha}{48EI} (-4\alpha^2 + 3)$ (**)
10		$(a \geq b)$ $\frac{Pb}{3EI\ell} \sqrt{\left(\frac{\ell^2 - b^2}{3}\right)^3}$	$\sqrt{\left(\frac{\ell^2 - b^2}{3}\right)}$	$x < a: \frac{Pbx}{6EI\ell} (\ell^2 - b^2 - x^2)$ $x = a: \frac{Pa^2b^2}{3EI\ell}$ $x > a: \frac{Pa(\ell - x)}{6EI\ell} (2\ell x - a^2 - x^2)$
11		$\frac{1}{9\sqrt{3}} \frac{M\ell^2}{EI}$	0,423l	$\frac{M\ell^2\alpha}{6EI} (\alpha^2 - 3\alpha + 2)$
12		$(a \geq 0,423\ell)$ $\frac{M}{3EI\ell} \sqrt{\left(\frac{\ell^2 - b^2}{3}\right)^3}$	$\sqrt{\left(\frac{\ell^2 - b^2}{3}\right)}$	$x < a: \frac{Mx}{6EI\ell} (\ell^2 - 3b^2 - x^2)$ $x > a: \frac{M(\ell - x)}{6EI} (x^2 + 3a^2 - 2\ell x)$

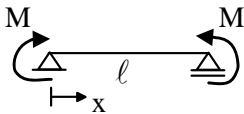
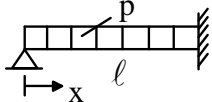
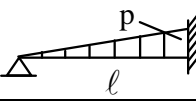
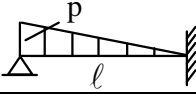
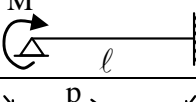
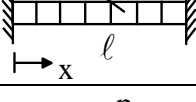
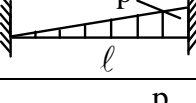

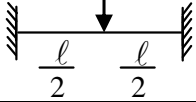
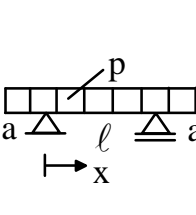
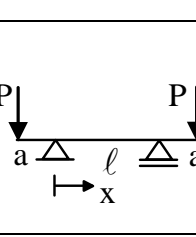
Extraída de ISNARD; GREKOW; MROZOWICZ (1971) e de SCHIEL (1976).

Revista e adaptada por Libânio M. Pinheiro, Bruna Catoia e Thiago Catoia.

$\alpha = x / \ell$

(*) Valor aproximado

(**) $\alpha \leq 0,5$

TABELA 3.1b				
DESLOCAMENTOS ELÁSTICOS EM VIGAS				
CASO	VINCULAÇÃO E CARREGAMENTO	FLECHA		EQUAÇÃO DA ELÁSTICA
		W_{max}	x	
13		$\frac{Ml^2}{8EI}$	$0,5l$	$\frac{Ml^2\alpha}{2EI}(1-\alpha)$
14		$\frac{3}{554} \frac{pl^4}{EI}$ (*)	$0,422l$	$\frac{pl^4}{48EI}(2\alpha^4 - 3\alpha^3 + \alpha)$
15		$\frac{3}{1258} \frac{pl^4}{EI}$ (*)	$0,447l$	$\frac{pl^4}{120EI}(\alpha^5 - 2\alpha^3 + \alpha)$
16		$\frac{1}{328} \frac{pl^4}{EI}$ (*)	$0,402l$	$\frac{pl^4}{240EI}(-2\alpha^5 + 10\alpha^4 - 11\alpha^3 + 3\alpha)$
17		$\frac{Ml^2}{27EI}$	$\frac{1}{3}l$	$\frac{Ml^2}{4EI}(\alpha^3 - 2\alpha^2 + \alpha)$
18		$\frac{1}{384} \frac{pl^4}{EI}$	$0,5l$	$\frac{pl^4}{24EI}(\alpha^4 - 2\alpha^3 + \alpha^2)$
19		$\frac{1}{764} \frac{pl^4}{EI}$ (*)	$0,525l$	$\frac{pl^4}{120EI}(\alpha^5 - 3\alpha^3 + 2\alpha^2)$
20		$\frac{7}{3840} \frac{pl^4}{EI}$	$0,5l$	$\frac{pl^4}{960EI}(16\alpha^5 - 40\alpha^3 + 25\alpha^2)$ (**)
21		$\frac{1}{192} \frac{Pl^3}{EI}$	$0,5l$	$\frac{Pl^3}{48EI}(-4\alpha^3 + 3\alpha^2)$ (**)
22		$\frac{pa}{24EI}(6a^2l + 3a^3 - l^3)$ $\frac{pl^2}{384EI}(5l^2 - 24a^2)$	$-a$ $0,5l$	$x < 0:$ $\frac{px}{24EI}(x^3 + 4ax^2 + 6a^2x + l^3 - 6a^2l)$ $0 < x < l:$ $\frac{px}{24EI}(x^3 - 2lx^2 + 6a^2x - 6a^2l + l^3)$
23		$\frac{Pa^2}{6EI}(2a + 3l)$ $-\frac{Pal^2}{8EI}$	$-a$ $0,5l$	$x < 0: \frac{Px}{6EI}(x^2 + 3ax - 3al)$ $0 < x < l: \frac{Pa}{2EI}x(x-l)$

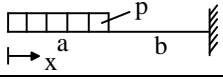

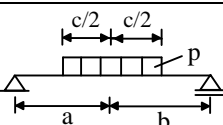
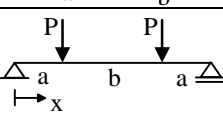
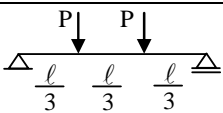
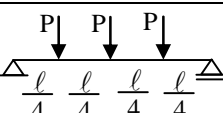
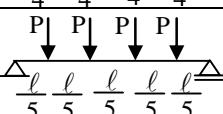
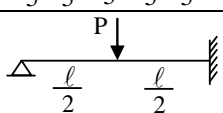
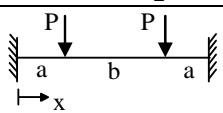
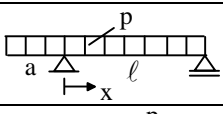
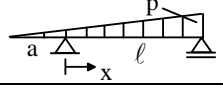
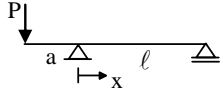
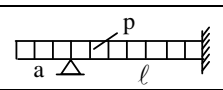
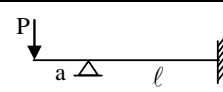
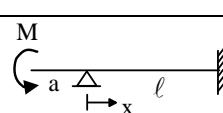
Extraída de ISNARD; GREKOW; MROZOWICZ (1971) e de SCHIEL (1976).

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$\alpha = x/l$

(*) Valor aproximado

(**) $\alpha \leq 0,5$

TABELA 3.1c			
DESLOCAMENTOS ELÁSTICOS EM VIGAS			
CASO	VINCULAÇÃO E CARREGAMENTO	FLECHA	
		W_{max}	x
24		$\frac{p}{24EI} (3l^4 - 4b^3l + b^4)$	0
25		$\frac{pa}{120EI} (20l^3 - 10al^2 + a^3)$	0
26		$\frac{pc}{6EI} \left[\frac{ab}{l} \left(2al - 2a^2 - \frac{c^2}{4} \right) \right] + \frac{c^3}{64}^{(*)}$	a
27		$\frac{Pa}{24EI} (3l^2 - 4a^2)$	0,5l
28		$\frac{23}{648} \frac{Pl^3}{EI}$	0,5l
29		$\frac{19}{384} \frac{Pl^3}{EI}$	0,5l
30		$\frac{63}{1000} \frac{Pl^3}{EI}$	0,5l
31		$\frac{\sqrt{5}}{240} \frac{Pl^3}{EI}$	0,447l
32		$\frac{1}{24} \frac{Pa^2b}{EI}$	0,5l
33		$\frac{pa}{24EI} (3a^3 + 4a^2l - l^3)$	-a
34		$\frac{pl}{360EI(a+l)} (20a^4 - 15a^2l^2 - 7al^3 + 12)$	-a
35		$\frac{Pa^2}{3EI} (a+l)$	-a
36		$\frac{pa}{48EI} (6a^3 + 6a^2l - l^3)$	-a
37		$\frac{Pa^2}{6EI} (4a+3)$	-a
38		$\frac{Ma}{4EI} (l+2a)$	-a



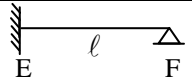
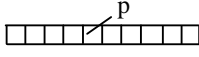
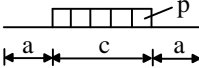
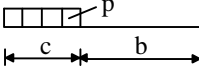
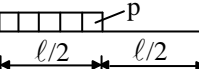
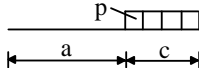
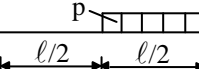
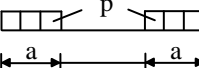
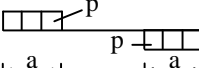
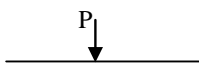
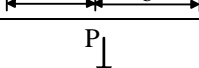
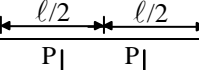
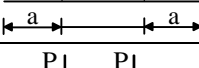
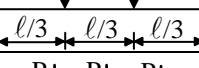

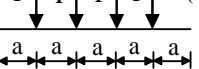
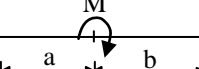
Extraída de ISNARD; GREKOW; MROZOWICZ (1971).

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(*) Não corresponde necessariamente ao deslocamento máximo

TABELA 3.2a

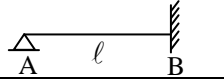
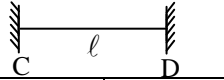
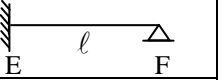
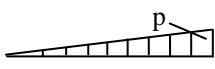
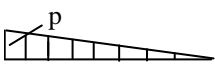
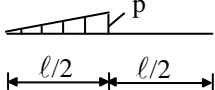
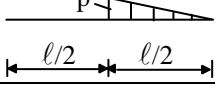
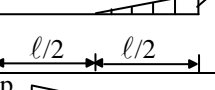
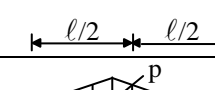
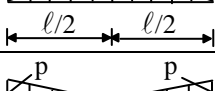
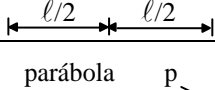
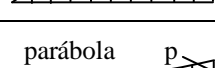
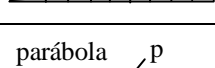
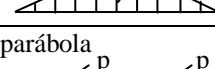
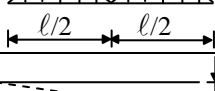
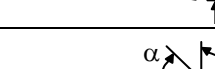
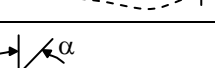
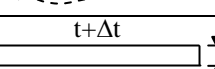

MOMENTOS DE ENGASTAMENTO PERFEITO

CARREGAMENTO				
	M_{BA}	M_{CD}	M_{DC}	M_{EF}
1 	$-\frac{p\ell^2}{8}$	$\frac{p\ell^2}{12}$	$-\frac{p\ell^2}{12}$	$\frac{p\ell^2}{8}$
2 	$-\frac{pc}{16\ell}(3\ell^2 - c^2)$	$\frac{pc}{24\ell}(3\ell^2 - c^2)$	$-\frac{pc}{24\ell}(3\ell^2 - c^2)$	$\frac{pc}{16\ell}(3\ell^2 - c^2)$
3 	$-\frac{pc^2}{8\ell^2}(2\ell^2 - c^2)$	$\frac{pc^2}{12\ell^2}(6b^2 + 4bc + c^2)$	$-\frac{pc^2}{12\ell^2}(4bc + c^2)$	$\frac{pc^2}{8\ell^2}(\ell + b)^2$
4 	$-\frac{7}{128}p\ell^2$	$\frac{11}{192}p\ell^2$	$-\frac{5}{192}p\ell^2$	$\frac{9}{128}p\ell^2$
5 	$-\frac{pc^2}{8\ell^2}(\ell + a)^2$	$\frac{pc^2}{12\ell^2}(4ac + c^2)$	$-\frac{pc^2}{12\ell^2}(6a^2 + 4ac + c^2)$	$\frac{pc^2}{8\ell^2}(2\ell^2 - c^2)$
6 	$-\frac{9}{128}p\ell^2$	$\frac{5}{192}p\ell^2$	$-\frac{11}{192}p\ell^2$	$\frac{7}{128}p\ell^2$
7 	$-\frac{pa^2}{4\ell}(3\ell - 2a)$	$\frac{pa^2}{6\ell}(3\ell - 2a)$	$-\frac{pa^2}{6\ell}(3\ell - 2a)$	$\frac{pa^2}{4\ell}(3\ell - 2a)$
8 	$-\frac{pa^2}{4\ell}(3\ell - 2a)$	$\frac{pa^2}{2\ell^2}(\ell - a)^2$	$-\frac{pa^2}{2\ell^2}(\ell - a)^2$	$\frac{pa^2}{4\ell}(3\ell - 2a)$
9 	$-\frac{Pab}{2\ell^2}(\ell + a)$	$\frac{Pab^2}{\ell^2}$	$-\frac{Pa^2b}{\ell^2}$	$\frac{Pab}{2\ell^2}(\ell + b)$
10 	$-\frac{3Pl}{16}$	$\frac{Pl}{8}$	$-\frac{Pl}{8}$	$\frac{3Pl}{16}$
11 	$-\frac{3Pa}{2\ell}(\ell - a)$	$\frac{Pa}{\ell}(\ell - a)$	$-\frac{Pa}{\ell}(\ell - a)$	$\frac{3Pa}{2\ell}(\ell - a)$
12 	$-\frac{Pl}{3}$	$\frac{2Pl}{9}$	$-\frac{2Pl}{9}$	$\frac{Pl}{3}$
13 	$-\frac{15Pl}{32}$	$\frac{5Pl}{16}$	$-\frac{5Pl}{16}$	$\frac{15Pl}{32}$
14 	$-\frac{Pl}{8n}(n^2 - 1)$	$\frac{Pl}{12n}(n^2 - 1)$	$-\frac{Pl}{12n}(n^2 - 1)$	$\frac{Pl}{8n}(n^2 - 1)$
15 	$-\frac{M}{2\ell^2}(\ell^2 - 3a^2)$	$\frac{Mb}{\ell^2}(3b - 2\ell)$	$-\frac{Ma}{\ell^2}(2\ell - 3a)$	$\frac{M}{2\ell^2}(3b^2 - \ell^2)$
16 	$-\frac{Pl}{16n}(2n^2 + 1)$	$\frac{Pl}{24n}(2n^2 + 1)$	$-\frac{Pl}{24n}(2n^2 + 1)$	$\frac{Pl}{16n}(2n^2 + 1)$

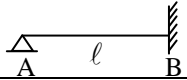
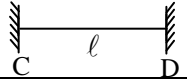
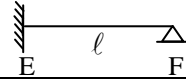
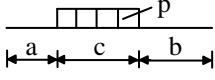
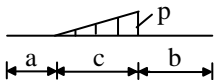
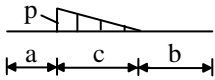
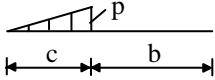
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(*) $n = \ell / a$

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TABELA 3.2b					
MOMENTOS DE ENGASTAMENTO PERFEITO					
CARREGAMENTO					
	M_{BA}	M_{CD}	M_{DC}	M_{EF}	
17		$-\frac{pl^2}{15}$	$\frac{pl^2}{30}$	$-\frac{pl^2}{20}$	$\frac{7pl^2}{120}$
18		$-\frac{7pl^2}{120}$	$\frac{pl^2}{20}$	$-\frac{pl^2}{30}$	$\frac{pl^2}{15}$
19		$-\frac{17}{480}pl^2$	$\frac{pl^2}{30}$	$-\frac{3}{160}pl^2$	$\frac{41}{960}pl^2$
20		$-\frac{41}{960}pl^2$	$\frac{3}{160}pl^2$	$-\frac{pl^2}{30}$	$\frac{17}{480}pl^2$
21		$-\frac{53}{1920}pl^2$	$\frac{7}{960}pl^2$	$-\frac{23}{960}pl^2$	$\frac{37}{1920}pl^2$
22		$-\frac{37}{1920}pl^2$	$\frac{23}{960}pl^2$	$-\frac{7}{960}pl^2$	$\frac{53}{1920}pl^2$
23		$-\frac{5}{64}pl^2$	$\frac{5}{96}pl^2$	$-\frac{5}{96}pl^2$	$\frac{5}{64}pl^2$
24		$-\frac{3}{64}pl^2$	$\frac{pl^2}{32}$	$-\frac{pl^2}{32}$	$\frac{3}{64}pl^2$
25		$-\frac{11}{120}pl^2$	$\frac{pl^2}{20}$	$-\frac{pl^2}{15}$	$\frac{pl^2}{12}$
26		$-\frac{pl^2}{24}$	$\frac{pl^2}{60}$	$-\frac{pl^2}{30}$	$\frac{pl^2}{30}$
27		$-\frac{pl^2}{10}$	$\frac{pl^2}{15}$	$-\frac{pl^2}{15}$	$\frac{pl^2}{10}$
28		$-\frac{7}{80}pl^2$	$\frac{7}{120}pl^2$	$-\frac{7}{120}pl^2$	$\frac{7}{80}pl^2$
29		$+\frac{3a}{l^2}EI$	$+\frac{6a}{l^2}EI$	$+\frac{6a}{l^2}EI$	$+\frac{3a}{l^2}EI$
30		$+\frac{3\alpha}{l}EI$	$+\frac{2\alpha}{l}EI$	$+\frac{4\alpha}{l}EI$	---
31		---	$-\frac{4\alpha}{l}EI$	$-\frac{2\alpha}{l}EI$	$-\frac{3\alpha}{l}EI$
32		$+\frac{3EI}{2h}\alpha_t\Delta t$	$-\frac{EI}{h}\alpha_t\Delta t$	$+\frac{EI}{h}\alpha_t\Delta t$	$-\frac{3EI}{2h}\alpha_t\Delta t$

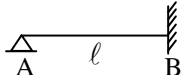
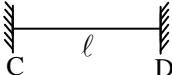
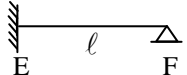
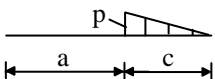
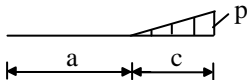
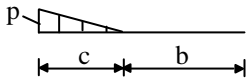
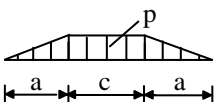
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TABELA 3.2c		
MOMENTOS DE ENGASTAMENTO PERFEITO		
CARREGAMENTO	  	
33		$M_{BA} = -\frac{P}{8\ell^2} \left[a^4 - (a+c)^4 + 2c\ell^2(2a+c) \right]$
		$M_{CD} = \frac{P}{12\ell^2} \left\{ 4\ell \left[(b+c)^3 - b^3 \right] - 3 \left[(b+c)^4 - b^4 \right] \right\}$
		$M_{DC} = -\frac{P}{12\ell^2} \left\{ 4\ell \left[(a+c)^3 - a^3 \right] - 3 \left[(a+c)^4 - a^4 \right] \right\}$
		$M_{EF} = \frac{P}{8\ell^2} \left[b^4 - (b+c)^4 + 2c\ell^2(2b+c) \right]$
34		$M_{BA} = -\frac{pc}{108\ell^2} (3a+2c) \left[9(\ell^2 - a^2) - 12ac - c^2 \left(4 + \frac{45a+28c}{30a+20c} \right) \right]$
		$M_{CD} = \frac{pc}{540\ell^2} \left[10(3b+c)^2(3a+2c) - 15c^2(3b-\ell) - 17c^3 \right]$
		$M_{DC} = -\frac{pc}{540\ell^2} \left[10(3b+c)(3a+2c)^2 - 15c^2(3a-\ell) - 28c^3 \right]$
		$M_{EF} = \frac{pc}{108\ell^2} (3b+c) \left[9(\ell^2 - b^2) - 6bc - c^2 \left(1 + 9 \frac{45b+17c}{270b+90c} \right) \right]$
35		$M_{BA} = -\frac{pc}{108\ell^2} (3a+c) \left[9(\ell^2 - a^2) - 6ac - c^2 \left(1 + 9 \frac{45a+17c}{270a+90c} \right) \right]$
		$M_{CD} = \frac{pc}{540\ell^2} \left[10(3a+c)(3b+2c)^2 - 15c^2(3b-\ell) - 28c^3 \right]$
		$M_{DC} = -\frac{pc}{540\ell^2} \left[10(3a+c)^2(3b+2c) - 15c^2(3a-\ell) - 17c^3 \right]$
		$M_{EF} = \frac{pc}{108\ell^2} (3b+2c) \left[9(\ell^2 - b^2) - 12bc - c^2 \left(4 + \frac{45b+28c}{30b+20c} \right) \right]$
36		$M_{BA} = -\frac{P}{30\ell^2} c^2 (5\ell^2 - 3c^2)$
		$M_{CD} = \frac{P}{30\ell^2} c^2 (10\ell^2 - 15c\ell + 6c^2)$
		$M_{DC} = -\frac{P}{20\ell^2} c^2 (5c\ell - 4c^2)$
		$M_{EF} = \frac{P}{120\ell^2} c^2 (40\ell^2 - 45c\ell + 12c^2)$

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TABELA 3.2d

MOMENTOS DE ENGASTAMENTO PERFEITO

CARREGAMENTO				
37		$M_{BA} = -\frac{P}{120\ell^2}c^2(40\ell^2 - 45c\ell + 12c^2)$		
		$M_{CD} = \frac{P}{20\ell^2}c^2(5c\ell - 4c^2)$		
		$M_{DC} = -\frac{P}{30\ell^2}c^2(10\ell^2 - 15c\ell + 6c^2)$		
		$M_{EF} = \frac{P}{30\ell^2}c^2(5\ell^2 - 3c^2)$		
38		$M_{BA} = -\frac{P}{120\ell^2}c^2(20\ell^2 - 15c\ell + 3c^2)$		
		$M_{CD} = \frac{P}{60\ell^2}c^2(5c\ell - 3c^2)$		
		$M_{DC} = -\frac{P}{60\ell^2}c^2(10a\ell + 3c^2)$		
		$M_{EF} = \frac{P}{120\ell^2}c^2(10\ell^2 - 3c^2)$		
39		$M_{BA} = -\frac{P}{120\ell^2}c^2(10\ell^2 - 3c^2)$		
		$M_{CD} = \frac{P}{60\ell^2}c^2(10b\ell + 3c^2)$		
		$M_{DC} = -\frac{P}{60\ell^2}c^2(5c\ell - 3c^2)$		
		$M_{EF} = \frac{P}{120\ell^2}(20\ell^2 - 15c\ell + 3c^2)$		
40		$M_{BA} = -\frac{P}{8\ell}(\ell^3 - 2a^2\ell + a^3)$		
		$M_{CD} = \frac{P}{12\ell}(\ell^3 - 2a^2\ell + a^3)$		
		$M_{DC} = -\frac{P}{12\ell}(\ell^3 - 2a^2\ell + a^3)$		
		$M_{EF} = \frac{P}{8\ell}(\ell^3 - 2a^2\ell + a^3)$		

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